Measuring the Ability to Exercise Unilateral Market Power in Locational-Pricing Markets: An Application to the Italian Electricity Market*

Christoph Graf†    Frank A. Wolak‡

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Abstract

An increasing number of wholesale electricity markets employ locational pricing mechanisms where energy prices account for some or all aspects of the transmission network configuration. A major concern of regulators is that suppliers may have the ability to exercise unilateral market power by impacting the extent to which transmission constraints bind. We extend the residual demand curve as a measure of the ability to exercise unilateral market power from a single price market to residual demand hyper-surfaces in locational pricing markets. We show that accounting for the fact that firms face residual demand surfaces improves our ability to explain the offer curves submitted by strategic suppliers. A supplier’s residual demand surface also explains why the location of a firm’s capacity is an important factor in analyzing the extent to which divestment of generation capacity or a transmission network expansion ultimately benefits final consumers.

Keywords: Locational pricing markets, Transmission network congestion, Unilateral market power

JEL Codes: C61, C80, D44, D47, L10, L22, L94

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†Department of Economics, Stanford University, Stanford, CA 94305-6072, cgraf@stanford.edu. Financial support from the Austrian Science Fund (FWF), J-3917, and the Anniversary Fund of the Oesterreichische Nationalbank (OeNB), 18306, is gratefully acknowledged.

‡Program on Energy and Sustainable Development (PESD) and Department of Economics, Stanford University, Stanford, CA 94305-6072, wolak@zia.stanford.edu
1 Introduction

All United States and an increasing number of international wholesale electricity markets employ mechanisms that explicitly incorporate the impact of transmission network constraints into the prices paid to specific generation units based on their location in the transmission network. More granular pricing of energy typically raises concerns about the exercise of local market power by generation units taking advantage of their location in the transmission network to increase their profits from selling energy at that location. This concern implies the need for methods for measuring the ability of a supplier that owns multiple generation units at different locations in the transmission network to exercise unilateral market power in a locational pricing market.

The purpose of this paper is to derive a general methodology for measuring the ability of suppliers that own generation units at multiple locations in the transmission grid to exercise unilateral market power in a locational pricing market. To this end, we extend the concept of a residual demand curve from a single price market to residual demand hyper-surfaces that account for the ability of a supplier that owns capacity at multiple locations to impact locational prices.\footnote{In a single market, unilateral market power may be exercised by withholding output. In locational markets, a firm with capacity in several locations may have several options to exercise market power. For example, increasing its output in one location may help to raise prices in another location. Our approach captures these incentives.} We then demonstrate the usefulness of this methodology for modeling supplier offer behavior and performing merger analysis and outline how it can be used in the transmission planning process in a locational pricing market.

Using this framework, we first quantify how the magnitude and location of supplier’s generation capacity impacts its expected profit-maximizing offer behavior in a locational pricing wholesale market. Depending on the locations of the firm’s generation capacity in the transmission network, the offer behavior at one location can impact the extent of competition the firm faces at another location. We apply this residual demand surface concept to model the best-reply offer behavior to the Italian zonal market and show the
importance of endogenous network constraints in explaining actual offer behavior. We also explore how allocating the same amount of generation capacity to different locations in the transmission network changes a supplier’s best-reply offer behavior and market-clearing outcomes.

There are a number of technical challenges associated with solving for the residual demand surface faced by a supplier in a potentially congested transmission network. First, it requires constructing a model of the actual price-setting process in the wholesale electricity market that replicates actual locational prices given the actual offer curves submitted by all market participants and produces counterfactual locational prices for any set of offer curves submitted by all market participants and any aggregate demand curve. Second, it requires solving an optimization problem to compute the expected profit-maximizing offers of a market participant where each function evaluation of the supplier’s expected profit function requires computing counterfactual market outcomes multiple times. We merge both problems into a single Mathematical Program with Equilibrium constraints (MPEC).

Applying the best-response offer problem to the Italian day-ahead market for electricity, we find that using expected-profit maximizing offer curves yields a 4% higher expected profit for the incumbent [8% higher for its main competitor] relative to the expected profit using the actual offer curves. In contrast, calculating best-response offer curves: (i) using the market-clearing process as it is, and (ii) assuming that there is infinite transmission capacity yields very different expected profit values. Using the best-response offer curve computed using the actual market clearing mechanism and actual transmission network constraints, we find that the firm earns higher expected profits than it would from using best-response offer curve computed ignoring transmission constraints in the market-clearing process.

To illustrate the importance of accounting for the configuration of the transmission network when modeling the behavior of strategic firms in a zonal market, we explore how expected zonal prices would change if part of a firm’s capacity is moved into another zone. In a single zone market, the location of a generation unit is irrelevant because units will
be dispatched based on the merit order for the entire market. In a transmission network with finite capacity, this is not necessarily the case because best-reply zonal prices depend on the zonal cost structure of the firm’s generation capacity and the zonal residual demand hyper-surfaces the firm faces. We find that at current locations of its generation units, the largest supplier in the Italian market withholds cheap coal capacity because it is locked-in in a zone where it has a significant ability to exercise unilateral market power. Moving this capacity to a zone where it faces greater competition from low-cost generation units causes it to offer more of this capacity at lower offer price which reduces the zonal price but also the load-share weighted average of zonal prices paid by the demand-side of the market. However, the average zonal price difference between the two zones would increase by 71% relative to the status quo. This exercise highlights the importance of supply side effects and their impact on zonal prices as well as transmission congestion.

For the purposes of assessing the competitive effects of mergers between generation unit owners, we demonstrate that the same amount of capacity divestment at different locations in the transmission network yields different counterfactual prices to load. More precisely, we find that divesting the same amount of capacity from the incumbent firm in two different locations would reduce the counterfactual price paid by load either by 7% or by 1%, depending on where the divestment took place. This result emphasizes the importance of accounting for the location of generation units in performing market power analyses in a locational pricing market.

We outline how residual demand hyper-surfaces can be used to assess the competitiveness benefits of transmission network upgrades. Specifically, a transmission expansion changes the form and distribution of the residual demand hyper-surfaces that a supplier faces which impacts the supplier’s expected profit-maximizing offers into the locational pricing market. This expansion can reduce the ability of a supplier to exercise unilateral market power at some of the locations that it owns generation units.

Finally, we believe our methodology can be used to design local market power mitigation
(LMPM) mechanisms for locational pricing markets. Local market power mitigation systems in the United States typically involve some sort of competitive transmission path analysis, (see, e.g., Rahimi et al., 2007). Key to this analysis is to identify suppliers that are pivotal for relaxing a binding (transmission) constraint. Practically, such form of locational market power may happen in supply-deficient areas such as large metropolitan areas with few local generation units, what are typically referred to as load pockets. A local supplier situated in a load pocket may find it profitable to withhold supply and therefore will congest the lines that connect the metropolitan area with the rest of the market yielding higher locational prices in that area. As we demonstrate below, high concentration of supply within load-pockets is only one of a number of potential opportunities to exercise market power in locational markets (see, e.g., Rahimi et al., 2007; Cardell et al., 1997).

The remainder of the paper is structured as follows. In Section 2, we extend the single-integrated-market residual demand concept to spatially-connected markets. In Section 3, we extend this concept to multi-unit auction markets. In Section 4, we introduce the best-reply model of offer behavior with endogenous congestion. After explaining how we solve for best-reply offer curves accounting for transmission network constraints in Section 5, we show how transmission constraints affect best-response offer curves and resulting market-clearing prices and quantities in Section 6. In Section 7, we present two counterfactual analysis. We first show how best-reply prices and quantities change when capacity owned by the incumbent firm is moved around the network. We then show the importance of a locational perspective when evaluating different divestment strategies. After outlining the effect of a transmission upgrade in Section 8, we conclude and suggest directions for future research in Section 9.

## 2 Residual Demand Surface Concept

This paper focuses on defining and exploring the properties of residual demand surfaces in multi-unit auction markets. To understand these concepts in a more familiar environment,
we first explain how the residual demand curve logic translates when moving from single price markets to spatially connected markets in a quantity-setting competition environment.

Assume a single market for a homogeneous good with $Q = D(P)$ being its demand function and $P = P(Q)$ its inverse demand function. Assume further that the action space of each firm is quantities. Given a fixed aggregate output of firm $i$’s competitors, $Q_{-i}$, firm $i$’s residual demand ($RD$) can be described as $RD_i(P) = D(P) - Q_{-i}$, and its inverse residual demand as $RD_i^{-1}(Q_i)$. While the residual demand gives the demand for firm $i$’s output at the market clearing-price, $P$, the inverse residual demand gives the resulting market clearing-price at a given level of the firm’s output $Q_i$.

We can easily extend this model to two or more spatially isolated markets where a firm owns generation capacity. The only difference will be that the firm would have separate residual demand curves in each market.

### 2.1 Spatially Connected Markets

The more interesting and relevant case is when spatial markets are connected and allow imports and exports up to the transport capacity constraints between the markets. We therefore consider a situation where two geographic markets are connected to each other. In the case of unlimited transport capacity between the two markets we can effectively study the single market case with the only difference that the two demand curves, one for each market, are added horizontally and also the aggregate supply of the competitors are added up. The residual demand a firm faces may be stated as $RD_i^{1,2}(P) = D^1(P) + D^2(P) - Q_{-i}^1 - Q_{-i}^2$.

These spatially distinct markets may be only partially connected because binding transport capacity limits the exchange between them, at least in the short run. This may be the case for many goods and services that need to be physically shipped between geographically separate markets. Even if shipping the goods was costless, finite transport capacity between the markets may affect the competitiveness of each local market. This framework applies to any homogeneous good produced and sold in spatially distinct markets where there are
limits on the quantity of the good that can be transported between the markets in a given period of time.

Borenstein et al. (2000) study an electricity market under the assumption of two local monopolists connected by a finite transport link facing the same local demand curve for the product. They find that a sufficiently large available transport capacity between the two markets increases the competition the supplier in each local market faces and even the existence of a sufficient quantity of available transport capacity will lead to more output produced by each local firm even though this transport capacity will be barely used in equilibrium. The consequence of the interconnection between the two markets is reflected in each firm’s residual demand curve. Each firm’s residual demand curve will consist of a part where both markets will be connected and parts where the transmission capacity will be exhausted and thus each supplier becomes a local monopoly.

We relax the assumption in Borenstein et al. (2002) that the firms operate in separate markets and explore what would happen to a firm’s residual demand function if it owns capacity in both markets. Therefore, we assume first, as in Borenstein et al. (2000), that both markets can be described by the same linear demand curve. Given a fixed aggregate output of firm $i$’s competitors in both markets, $Q_{1-i}, Q_{2-i}$, firm $i$ may be able to congest the fixed and symmetric transportation capacity, $K$, by its choice of outputs, $Q_i^1, Q_i^2$. In Figure 1, we depict an example of such markets. The relevant price for Market 1 (M1) is on the left vertical axis and the price for Market 2 (M2) on the right vertical axis in each of the two panels. The total output in M1 is on the horizontal axis read from the left to the right and the total output in M2 is on the same axis read from the right to the left. The figures in Panel (a) and (b) show how the two interconnected markets would clear given a fixed amount of transport capacity between the two markets, and a given amount of zonal outputs, $Q^1 = Q_i^1 + Q_{1-i}, Q^2 = Q_i^2 + Q_{2-i}$. The fixed and symmetric transport capacity, $K$ is indicated by the line between the zonal equilibrium demand levels and the zonal output. In Panel (a), we see that at the given zonal output of firm $i$, the network will be uncongested
as the resulting export from M1 to M2, \( r \), or equivalently the resulting import from M2 to M1, \(-r\), does not exceed the physical transport capacity limit, \( K \). Although, the transport capacity between the two markets is uncongested, some output in M1 is used to serve demand in M2. Holding the total amount of output constant, one can, by varying the zonal output of firm \( i \), see that for many combinations of locational outputs the transport capacity between the two markets would essentially be uncongested. This changes, however, if firm \( i \) increases its output substantially in M1 as depicted in Panel (b). Firm \( i \)'s oversupply in M1 and its undersupply in M2 leads to congestion of the transport capacity since not all the supply of M1 can be exported to M2. As a result of the congested transport capacity, prices in the two markets will differ.

Whether the transportation network will be congested for a given level of outputs, exogenous market demand curves, and a fixed transport capacity connecting the two markets can be determined by an optimization problem that maximizes aggregate welfare, i.e., the sum of the areas below the two locational demand curves by implicitly accounting for resulting trade in these markets up to an extend where the market prices will be the same. Furthermore, a capacity constraint that limits the possible amount of trade between the markets is added to the optimization problem. One way to formulate such a problem for the two-markets case is

\[
J^*(Q^1, Q^2) = \arg \max_r \quad \int_0^{Q^1-r} P^1(\tau_1)d\tau_1 + \int_0^{Q^2+r} P^2(\tau_2)d\tau_2 \tag{1a}
\]

subject to

\[
-K \leq r \leq K, \tag{1b}
\]

\[
Q^i + r \geq 0, \quad i \in \{1, 2\}. \tag{1c}
\]

Problem 1 can be solved for any vector of zonal aggregate output, whose elements are \(Q^1\) and \(Q^2\), resulting in quantities consumed in each market and also their locational prices. These prices will be equal for both markets if the constraint in (1b) is not binding and will differ otherwise. The formulation of the nodal pricing model (1) yields a functional mapping
from the aggregate output to prices in the two markets \( f: \mathbb{R}^2_+ \rightarrow \mathbb{R}^2_+ \). Holding \( Q^1_i \) and \( Q^2_i \) fixed, we can solve it for any vector of firm \( i \)'s zonal output. In electricity markets, we assume transport is costless but the model is easily extendable to connected markets with a non-zero transportation cost. In the case of a per unit charge to move the product between markets equal to \( c \), the term \( c |r| \) must be added to the objective function (1a). The consequence of a transportation cost is that less trade will occur.

In the next step, we show how the residual demand concept can be extended to markets where firms have capacity in several markets together with endogenous congestion power. We will use the solutions of the optimization problem described above, solved for many locational output combinations of one firm fixing the outputs of its competitors. Remember that unlike in Borenstein et al. (2000), we are dealing with a case where the firm is operating in both markets, hence its output is always a vector. Furthermore, its output in M1 and M2 together will determine whether the transport capacity between the markets will be congested. It is true, though, that even if the firm had capacity in only one market it could congest the transport capacity as discussed in Borenstein et al. (2000). However, having capacity available in both markets may increases the firm’s power to congest the transport capacity as it can set the output in both markets instead of only in one market. This becomes evident by looking at Figure 1, Panel (b). It could be possible that a firm finds it unilaterally profitable to set a high output in M1 even when the firm would not be able to recover its zonal cost at such a level of output. It does this by congesting the network and increasing its revenue from sales in M2. In the case discussed in Borenstein et al. (2000), the firm’s zonal residual demand curve is partially replaced by the aggregated market residual demand curve for the output range that would lead to a non-binding transport capacity constraint. Consequently, the points where the slope of the firm’s residual demand curve changes for the Borenstein et al. (2000) model will be solely determined by the firm’s zonal output for a given transport capacity limit and the output of the firm’s competitors. This is no longer the case when accounting for firms with capacity in two connected markets.
As described above, the optimization problem described in (1) can be solved for many vectors of the firm’s output and gives corresponding zonal market-clearing prices. Hence, there is a functional relationship between the firm’s output vector and the market-clearing price vector, which can be used to construct a residual demand surface, one for each of the two connected markets where the firm owns capacity. Considering that the firm has capacity in both markets makes visualizing the relationship between the firms output and prices more complex, as the output of the firm is a vector. One way of visualizing this relationship is to plot zonal outputs on the horizontal axes \((x, y\text{-axes})\) and the zonal price on the vertical axis \((z\text{-axis})\). We refer to each of these graphs as the inverse residual demand surface and for the case of two connected markets there will be two of these surfaces.\(^2\) Although, we choose to plot each surface in a separate figure they are meant to be read together, as the gradients are only comparable at a specific point in the \((Q^1_t, Q^2_t)\)-space. In Figure 2, we visualize the residual demand surfaces corresponding to the example depicted in Figure 1. Panel (a) shows the inverse residual demand surface for M1 and Panel (b) for M2. Remember that every point on the surfaces in Panel (a) and Panel (b) for the same level of \(Q^1_t\) and \(Q^2_t\) are the result of the optimization problem in (1). In Panel (c), we show the price difference as a function of outputs. This figure may be interpreted as the unilateral endogenous power to congest the network. For all output combinations leading to uncongested markets (all the points where \(f(Q^1_t, Q^2_t) = 0\) in Figure 2, Panel c), the residual demand surface is effectively the residual demand curve for the uncongested single price market. However, it is important to notice that whether there is an uncongested market or a congestion between markets depends on the firm’s output in each market, as is evident from Figure 2.

The residual demand surfaces depicted in Figure 2, Panel (a) and (b), are not symmetric as the aggregated zonal supply of firm \(i\)'s competitors differ for each market. We eschew the discussion of a case where both markets were identical and the levels of aggregate output of

\(^2\)We go with the convention to plot the price on the vertical axis (\(z\text{-axis})\). Hence, we plot the resulting zonal market clearing-price as a function of the firm’s outputs in both markets. When we refer to a pair of inverse residual demand curves—one for each zone—we denote it by residual demand surfaces as they should be read together.
the firm’s competitors in each zone too. In this case, the residual demand surfaces would be symmetric but there is no point in doing this exercise as there is absolutely no structural difference between the two markets and consequently there will not be any room to unilaterally exploit it. However, an interesting and also more realistic case is where the two markets do not only differ in the way the firm’s opponents supply but also with respect to the steepness of the demand curves. We depicted such a scenario with two particular market equilibria for two specific output vectors of the firm in Figure 3, Panel (a) and Panel (b). The resulting residual demand surfaces are depicted in Figure 4. Notice that the only modification in comparison to Figure 3 is the change in the slope and intercept of M1. However, as evident from the comparing Panels (b) in Figures 1 and 3, this change impacts also the form of the inverse residual demand surface of M2. Another straightforward exercise would be to change the level of the transport capacity, $K$. If the transport capacity between the markets was reduced, the firm has a greater ability to unilateraly congest it.

We want to emphasize that the residual demand surface, as we define it, is simply the demand for a firm’s output in each zone at a pair of zonal prices, or equivalently, the resulting zonal prices for a given pair of the firm’s zonal output. The elegance of this concept is that it allows us to incorporate a transport capacity constraint that partially connects these markets. Therefore, it is a tractable concept that allows us quantify a firm’s ability to exercise locational unilateral market power (which includes the ability to congest the transport network). Unlike in the single price market, both the own-price elasticity of the zonal residual demands and the cross-price elasticities of the zonal residual demands will determine the optimal offer behavior of a supplier. Note that the residual demand surface is a hybrid between firm’s aggregate residual demand curve in a fully connected market and the firm’s zonal residual demand curves shifted by the resulting import, respectively, export. However, since the import/export flow is endogenously determined, the corresponding switching points between unconstrained and constrained markets depends on the output combinations chosen by the firm.
3 Residual Demand Concept in Multi-Unit Auction Markets

We now extend the residual demand surface concept to electricity supply industries. For simplicity, we assume the demand for electricity is perfectly price inelastic. Extending to the case of elastic demand at each location is straightforward, because the elastic demand case can be converted into an inelastic demand case by recognizing that a price responsive demand $D(p)$ can be re-written as an inelastic demand of $D(0)$, demand at zero price, and a price-responsive supply of “nega-watts” of $SN(p) = D(0) - D(p)$.

Because electricity markets are organized as multi-unit auction markets, it is possible to derive a residual demand function for each strategic firm. If transmission constraints were absent, this function is simply $RD_i(p) = D - S_{-i}(p)$ for each firm $i$, that is, the inelastic market demand, $D$, minus the aggregated offer curve of firm $i$’s opponents $S_{-i}(p)$. Note the case with a price-responsive demand $D = D(0)$ and $SN(p)$ is included in $S_{-i}(p)$. Because the offer curves are restricted to be upward slopping this results in a downward sloping residual demand curve. In a single-price market, the market-clearing price is the solution in $p$ to the equation $RD_i(p) = S_i(p, \theta_i)$, where $S_i(p, \theta_i)$ is firm $i$’s aggregate supply curve that is a function of the price as well as the parameters of its step function offer curve $\theta_i$. As discussed in Appendix C, $\theta_i$ is composed to offer price and incremental quantity offers that make supplier $i$’s offer curve. Because the residual demand curve firm $i$ is facing depends on the offer curves and demand bids submitted by all other market participants,
the market-clearing price depends on the vector of bid parameters of all market participants, $\theta$.

In a multi-zone network, the market-clearing process is more complex since network flows have to be accounted for. The market operator or transmission system operator manages these flows with the objective to maximize overall consumer’s plus producers’s surplus based on the offers submitted by suppliers and bids submitted by retailers and large consumers. However, a firm may take the operator’s response into account when offering its capacity to the day-ahead market. Consequently, a firm’s residual demand function becomes a mapping from the prices in different zones, the firm owns capacity in, to the demand for output from the firm in each zone at this vector of prices. Because of transmission network constraints, the zonal price does not only depend on a firm’s supply in this zone but also on its supply in other zones it is active in. More formally, the residual demand curve ($f: \mathbb{R}_+^1 \rightarrow \mathbb{R}_+^1$) in a single price market turns into a hyper-surface ($f: \mathbb{R}_+^Z \rightarrow \mathbb{R}_+^1$) for each of the $Z$ zones in which the firm owns generation capacity. For example let us assume that a firm operates in a two-zone network possessing available capacity in both zones. It follows that the firm faces a residual demand surface for each of the two zones, $Q_1^1(p_1, p_2)$ and $Q_2^2(p_1, p_2)$. Hence, the residual demand surface maps prices in the two zones to zonal quantities demanded from the firm at these two prices.

In the case of the Italian zonal day-ahead market for electricity, the market-clearing engine described in Appendix C can be used to compute residual demand hyper-surfaces by clearing the market conditional on possible zonal output values for a strategic firm (see Appendix D for more details).

In Figure 5, we show such residual demand surfaces. It is clear, that if the firm operates only in one of the two zones its residual demand functions would be a one-dimensional surface, i.e., a curve. It is also clear that in higher dimensions, the residual demand surfaces will become residual demand hyper-surfaces and are hard to visualize and best-reply offer curves will be harder to compute. However, in many cases electricity markets are composed
only of a small number of pricing zones. In many other cases, e.g., in classical locational marginal pricing markets as seen throughout the US, the network topology reduction to a tractable number of zones can be a reasonable approximation. In Appendix C.5, we show that in the case of the Italian transmission network which consists of seven domestic zones in the year 2007, a three-zone model provides an adequate representation of market outcomes. Appendix C and D discuss the details of the computation of residual demand surfaces for our three-zone—North, Center, and Sicily—model of the Italian electricity market.

We classify the residual demand surfaces\(^3\) into three main categories. First, the inverse residual demand surface can be unresponsive to changes in quantities in both zones the firm has capacity in. The inverse residual demand surface would therefore be a horizontal plane implying that the firm has no market power at all. This is the equivalent case to a flat inverse residual demand curve in a single market.

Second, the inverse residual demand surface can be unresponsive to changes in the firm’s output in one zone but responsive to changes in the firm’s output in the other zone. In such a case the firm has market power in one zone that may also include congestion power. In Figure 6, Panel (a) and (b), for example, we show the inverse residual demand surfaces for two zones, North and Center while in Panel (c) we show the load-weighted average of the zonal prices. The inverse residual demand surface in the Center (Panel b) is completely flat, meaning that the price will be unaffected by the firm’s scheduling decision in both zones. The inverse residual demand surface in the North (Panel a), however, is unresponsive only for small quantities in the same zone but becomes more responsive thereafter. Hence, the firm has market power in this zone. However, the firm has also congestion power as the zonal price difference increases as more power will be supplied in the North. In this particular case, congestion between the North and the Center will increase as the firm supplies more in the North. We will elaborate on this issue in the next subsection.

Third, the inverse residual demand surface is responsive to the firm’s zonal output in both

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\(^3\)In this paragraph, we focus on a two zone model as we are referring to residual demand surfaces and not hyper-surfaces. However, all our arguments translate also to networks with more than two zones.
zones. In this case zonal prices and also congestion are influenced by the output of the firm in both zones jointly. In Figure 5, Panel (a) and (b), we show the residual demand surfaces of a firm for two zones (North and Center) and in Panel (c) the load-weighted average price of the zonal prices. The firm has capacity in both zones and notably substantial market power in the Center depicted in Panel (b). Especially, the inverse residual demand surface in the North (Panel a), is very sensitive to supply in both zones. Furthermore, we see that congestion is likely to occur if the firm is withholding capacity in the Center.

The information contained in the Panels (a) and (b) in Figure 7 is comparable to that in Panels (a) and (b) in Figure 5 with the difference that in Figure 7, we plot zonal inverse residual demand curves conditional on different zonal supply levels of that firm in other zones. This representation has the advantage that even more than two-zone markets can be visualized. In Figure 7, Panel (c) we added a third market zone. As shown in the graphs, residual demand surfaces can be complex highly non-linear objects embodying significant (cross price) elasticities that imply the locations of generation capacity in the transmission network impacts the ability of a supplier to exercise unilateral market power in a local market.

In order to stress the importance of the residual demand surface concept, Figure 8 demonstrates how a firm’s zonal residual demand curve changes in response to changes in demand in another zone. In both Panels, we observe that also the slopes of the zonal inverse residual demand curves change as a response to demand changes in another zone. This aspect would be missed if either only individual zonal residual demand curves were analyzed or the system residual demand curve.

[Figure 5 about here.]

[Figure 6 about here.]

[Figure 7 about here.]

[Figure 8 about here.]
3.1 Unilateral Market Power and Endogenous Congestion Power

In Figure 6, Panel (a) and (b), we show an example of a firm with unilateral market power including endogenous congestion power only in the North (Panel a). As discussed in the previous section the firm may choose to withhold capacity in order to have the network *uncongested* and would therefore benefit from a higher zonal price which will be closer to the price in the high-price zone (Panel b). A simple way of visualizing the supply levels where the network will be congested, is to overlap the two inverse residual demand surfaces in Figure 6, Panel (a) and (b), or equivalently, take the difference between the zonal prices as we do in Figure 9. The zonal supply combinations where the price difference is zero correspond to an uncongested network. In Panel (b), we show that if this firm were to schedule only a small quantity in the North the price difference between the North and the Center will be zero, hence, the network will be uncongested. In Figure 9, Panel (a), we repeat this exercise for another firm and find that for large quantities in the Center the network will be uncongested. Hence, the firm’s capacity distribution along the network together with its spatial cost structure will determine whether a firm finds it profitable to unilaterally congest the network.

[Figure 9 about here.]

3.2 Neglecting Transmission Constraints

Figure 10 shows the market clearing mechanism in two zones accounting for network flows using actual offer data. This is a case where the transmission flow constraint between the North (Panel a) and the Center (Panel b) is binding and thus zonal prices are different. If we were to ignore network constraints in clearing the market, the same market would look as in Figure 11. As is evident from the figure, the market price would be closer to the price in the Center. This outcome emphasizes the fact that a firm with the ability to exercise unilateral market power in only one zone as depicted in Figure 6 would offer to
supply energy differently if it did not account for transmission constraints in determining its expected profit-maximizing offer curve. In Figure 19, we show that the firm’s residual demand curve ignoring the transmission network, would be almost flat. As a consequence, the firm would offer more aggressively, i.e., closer to its marginal cost, if it did not account for transmission constraints\(^4\) because its perceived residual demand would be more elastic than it is when accounting for transmission constraints.\(^5\)

[Figure 10 about here.]

[Figure 11 about here.]

[Figure 12 about here.]

4 Optimal Offers in a Locational Pricing Market

This section presents our model of optimal offer behavior in a locational-pricing market. This model respects both the markets rules on the set of feasible offer curves that all suppliers can submit and the actual locational-pricing market-clearing mechanism to compute the zonal prices that result from a given offer curve and each residual demand curve realization.

Competition in a wholesale electricity markets is typically modeled using supply functions with demand uncertainty as described in Klemperer and Meyer (1989). However, there are at least three reasons why this theoretical framework may not adequately represent actual outcomes in short-term wholesale electricity markets. First, participants in these markets actually submit non-decreasing step functions with a finite number of price and quantity steps. As noted in Wolak (2003) and Wolak (2007), this requirement invalidates the approach used by Klemperer and Meyer (1989) to compute a firm’s expected profit-maximizing offer curve as the envelope of best-reply price-quantity pairs. The Klemperer

\(^4\)We refer to Section E for a discussion of cases where neglecting transmission constraints will not reduce the slope of the residual demand a supplier faces.

\(^5\)In the best-response offer context a firm would be indifferent between offering at marginal cost or at any other offer curve that is between the marginal cost offer curve and an offer curve that is epsilon below the perfectly elastic residual demand curve.
and Meyer (1989) approach implies that the supplier’s expected profit-maximizing supply curve does not depend on the distribution of residual demand uncertainty. However, as shown in Wolak (2003) and Wolak (2007), under a market rule that requires suppliers to submit non-decreasing step functions, a supplier’s expected profit-maximizing offer curve generally depends on the distribution of step-function residual demand curves that it faces.

A second reason arises even if the supply functions are required to be continuous increasing functions as modeled in, e.g., Green and Newbery (1992); Sioshansi and Oren (2007); Anderson and Hu (2008): Transmission constraints cause expected profit-maximizing offer curves to depend on the probability distribution of random shocks to demand and transmission capacity, and the equations to be solved are highly nonlinear as shown in Wilson (2008).

The economics literature on optimal offer behavior in multi-unit auction markets, such as wholesale electricity markets, has largely ignored the impact of modeling the impact of transmission constraints in the pricing of energy (Wolak, 2000, 2007; Hortalçu and Puller, 2008; Reguant, 2014). We extend the model in (Wolak, 2000) to the case of a zonal-pricing model with a potentially congested transmission network. The best-reply offer model is applied to a net-selling (potentially vertically-integrated) firm $i$ operating in a locational marginal pricing market with a quantity-weighted average price paid by load and with each generation unit being paid its locational marginal price for its energy production.

We begin by introducing the notation necessary to present our model. Each firm $i = 1, \ldots, N$ operating in the market, offers a vector of price and quantity increments to the market expressing its willingness to supply energy from its generation units. Let $\theta_i = \{b_i, g_i\} = (b_{i,1}, \ldots, b_{i,K}, g_{i,1} \ldots, g_{i,K})$ be firm $i$'s vector of offer of prices and quantities.

---

$^6$We focus on a radial transmission network topology. In such a network loops are absent meaning that there is a unique path between every two nodes. In a radial transmission network the effect of congestion on a transmission link is to separate the system into zones at either end of the link, each zone having its own price for energy. This implies that each zone has an induced net demand for energy, i.e., the original zonal demand plus prescribed exports minus prescribed imports as shown in Wilson (2008). The market operator determines imports/exports between zones based on minimizing the as-offered costs to serve demand and subject to the physical capacity limits of the transmission lines.
It contains pairs of prices and quantities for each generator $j = 1, \ldots, J$ and each possible step $k = 1, \ldots, K$. Furthermore, define $\theta = \theta_i \cup \theta_{-i} \cup \theta_b$, the vector of offers of all market participants, where the subscript $b$ denotes demand side bids and subscript $-i$ the supply side offers of $i$’s opponents. The elements of $\theta_i$ make up a non-decreasing step function offer curve and the elements of $\theta_b$ make up non-increasing step function bid curves for demand.

We now define the firm’s realized profit function $\Pi_i(\theta_i)$. Define $S_{iz}(p_z, \theta_i)$ as the willingness-to-supply of firm $i$ at location $z$. Furthermore, let $p_z(\theta)$ be the short term price at location $z$ and $\overline{p}(\theta)$ the wholesale energy purchase price for loads. For the remainder of the paper, the purchase price for loads is assumed to be the load-share-weighted-average of zonal prices. Load paying according to this price is a feature of the Italian market that is the focus of our empirical analysis.\footnote{See Appendix C.1 for a more detailed description on the computation of the uniform purchase price. In all locational pricing markets in the US, load-serving entities typically pay for energy according to quantity-weighted averages of locational prices. Tangerás and Wolak (2018) demonstrate that this market rule can enhance the competitiveness of short-term market outcomes.} However, having loads pay the price in their congestion zone would be a straight forward modification of the model.

We denote the firm’s fixed retail price as $P^R$, its retail load obligation as $Q^R_i$, and the marginal cost of electricity retailing as $\tau_i$. The variable cost of output level $q$ at location $z$ is denoted as $C_{iz}(q)$. Finally, let $PC_{iz}$ the quantity-weighted average price of fixed-price forward contracts that settles against the uniform purchase price and $QC_{iz}$ the net quantity of fixed-price forward contracts for location $z$.

In terms of this notation, the realized variable profit function for a vertically-integrated firm is\footnote{Note that the zonal price is determined endogenously but in order to make the notation simpler we write $p_z$ instead of $p_z(\theta)$. The wholesale purchase price for load $\overline{p}$ is the load-share weighted average of zonal prices.}

$$\Pi(\theta_i) = (P^R - \overline{p})Q^R_i + \sum_z S_{iz}(p_z, \theta_i)p_z - \sum_z (\overline{p} - PC_{iz})QC_{iz} - \tau_i Q^R_i - \sum_z C_{iz}(S_{iz}(p_z, \theta_i)). \quad (2)$$

The first term is the variable profit from selling retail electricity. The second term is the revenue from wholesale electricity sales in the short-term market. The third term is total
difference payments associated with settling the suppliers fixed-price forward contract obligations. The fourth term is variable cost of electricity retailing. The final terms is variable cost electricity sold in the short-term market. $PC_{iz}$ is equal to

$$PC_{iz} = \frac{\sum_k PC_{izk} QC_{izk}}{QC_{iz}},$$

where $QC_{iz} = \sum_k QC_{izk}$, and $PC_{izk}$ is the price of forward contract $k$ at location $z$ and $QC_{izk}$ is the net forward position of contract $k$ at location $z$. A contract sold by the supplier is a positive value of $QC_{izk}$ and a contract purchased by the supplier is a negative value of $QC_{izk}$.

Each firm in the market is assumed to maximize expected profits conditional on the distribution of beliefs about its competitors’ offer strategies and demand side bids $\theta \setminus \theta_i$. Market-clearing prices and quantities are determined by the clearing of a transmission network-constrained auction with a uniform purchase price for loads but potentially different prices paid to generation units in each congestion zone.\(^9\) Formally, the firm’s expected profit-maximization problem is:

$$\begin{align*}
\text{maximize}_{\theta_i} & \quad \mathbb{E}_{\theta \setminus \theta_i} [\Pi_i(\theta_i)] \\
\text{subject to} & \quad 0 \leq b_i \leq \hat{b} \\
& \quad 0 \leq \sum_k g_{i,jk} \leq \hat{g}_{i,j}, \forall j
\end{align*}$$

where $\hat{b}$ is the offer price cap set by the regulator and $\hat{g}_{i,j}$ is the capacity constraint on generation unit $j$ owned by firm $i$.

Note that the value of $\theta_i$ that maximizes the firm’s expected profits over possible residual

\(^9\)A detailed explanation on how the market clears can be found in Section 3 and Appendix C.
demand realizations maximizes the expected value the following function

$$\Pi(\theta_i) = -\bar{p}Q^R + \sum_z S_{iz}(p_z, \theta_i)p_z - \bar{p} \sum_z QC_{iz} - \sum_z C_{iz}(S_{iz}(p_z, \theta_i)).$$  (4)

This expectation is taken with respect to the distribution of residual demand hyper-surfaces faced by the firm. Each residual demand hyper-surface realization gives rise to potentially different zonal prices and quantity-weighted average of the zonal prices, $\bar{p}$ that depends on the value of $\theta_i$. Therefore, the term $\bar{p}Q^R$ does matter to the firm’s expected profit-maximizing value of $\theta_i$.

Our model is able to accommodate different firm and market structures. If a firm is a generation company only and has no load to serve, $Q^R_i$ is set to zero. If transmission constraints were never binding, and thus $p_z = \bar{p} = p, \forall z$, the model translates to that described in Wolak (2000).

The model is also flexible in terms of different market structures. The realized profit function in (4) is valid for markets organized as a “gross-pool”. This means that firms offer all their capacity to the market and the aggressiveness of their offer behavior is determined by their forward commitments. However, only a small modification is necessary to accommodate “net-pool” markets, where firms have the option to physically schedule (part of) their bilateral forward commitments which are financially settled outside the market.\(^\text{10}\) This is the market structure that exists in Italy. Hence, we adapt (4) as follows:

$$\Pi(\theta_i) = -\bar{p}Q^R_i + \sum_z S_{iz}(p_z, \theta_i)p_z - \bar{p} \sum_z QC_{iz} + \sum_z QP_{iz} \delta_z - \sum_z C_{iz}(S_{iz}(p_z, \theta_i) + QP_{iz}^S).$$  (5)

In this case, $Q^R_i$ and $S_{iz}$ are net of the physical bilateral schedules. Furthermore, the

\(^{10}\)From a firm’s risk perspective, a “gross-pool” is equivalent to a “net-pool” in a single-pricing-zone market. However, this is not necessarily the case in a market with multiple pricing zones, a uniform purchase price for loads, and vertically-integrated firms. By self-scheduling supply capacity as well as load in the same zone, a vertically integrated firm can avoid to being exposed to the uniform purchase price risk (we provide more details and an example on this point in Section B.1).
$Q_{P,z}$ are the zone-level net physical bilateral schedules and $Q_{P,z}^S$ are the zone-level physical bilateral supply schedules. For scheduled bilateral transactions explicit transmission usage charges are due. They are calculated as the zonal sum of $\delta_z = \bar{p} - p_z$ multiplied with the net-injected zonal quantity.$^{11}$

5 Computing Expected Profit-Maximizing Offer Curves

In this section, we show how to compute expected profit-maximizing offer curves given the distribution of residual demand surfaces faced by the firm. This requires choices for the main modeling inputs necessary for the analysis such as generation unit-level variable costs, the residual demand surface distribution, and the fixed-price forward contract positions held by suppliers.

Our sample spans from September 1, 2007 to November 1, 2007.$^{12}$ We only optimize over peak hours, which are hours 11, 12, 18, 19, and 20. We eliminated weekends and holidays from the sample.

Our main reasons for selecting this time period are the following: (i) hydro production usually peaks in spring after the snow-melt and thus its influence on the market is reduced; (ii) a relatively stable amount of imports from France and Switzerland; and (iii) electricity production from intermittent renewable sources, such as wind or the sun was negligible at this time.

$^{11}$Note that $\delta$ is endogenously determined through $p_z$. Note also that the transmission usage charges can also be positive, depending on the price difference and the zonal net injection value. Put differently, a net producer pays the congestion cost if located in an exporting zone where the zonal price is smaller than the uniform purchase price. On the contrary, a net supplier in an importing zone receives the fee from the transmission system operator.

$^{12}$On the following dates, we observed missing values for the zonal assignment of bids: 2007-09-01, 2007-09-09, 2007-09-28, 2007-10-19, 2007-10-21, 2007-10-26, and 2007-10-28. We therefore, deleted these days from our sample. We further delete the following set of days from our sample due to a restricted line capacity between N and CN: 2007-09-16, 2007-10-07, 2007-10-22, 2007-10-23, 2007-10-24, 2007-10-25, 2007-10-26, 2007-10-27, 2007-10-28. The average peak-load line capacity was about 2,500 MW between the two zones but was below 1,700 MW on these days. We assume that this has an effect on the offer behavior because transmission capacity limits are known to the market participants before they submit their offer curves to the day-ahead market. Hence, including scenarios with substantially different transmission capacity limits would bias our results.
We have selected two strategic firms, Enel and Edison, which we refer to as the incumbent and the main competitor. We calculate best-response offer curves for each firm separately assuming that each firm maximizes expected profits given the distribution of residual demand hyper-surfaces that it faces. In this paper, we focus on the day-ahead market as this is the most important one in terms of traded quantities and furthermore its weighted average clearing price is the reference price for forward contracts and serves as a signal for investment. A detailed description of the Italian electricity supply industry can be found in Appendices B.1 and B.2.

We optimize only over offer prices of relevant thermal units with price offers larger than their estimated variable cost of production. We consider a thermal unit to be relevant if it is optimizing its offers to participate in the day-ahead market.\textsuperscript{13} The criteria we define to determine whether a unit is relevant is the share of day-ahead market sales relative to final production. Thermal units below a 50\% share or above a 150\% share are not treated as strategic.\textsuperscript{14} This criteria selects 60\% of the thermal capacity of our two selected strategic firms.

Our approach of fixed the offer quantities at their actual values would be problematic if significant physical capacity withholding were present in the Italian market. However, the coefficient of variation of total hourly offered quantity over the sample period is only 5\% in the case of the incumbent and 10\% in the case of the main competitor. Because generation units are taken out of service for maintenance and unplanned outages, these numbers are consistent with our expectations for the impact of these outages and therefore make us confident that these market participants maximize expected profits by making all of their available capacity to the short-term market and alter their offer prices in response to competitive conditions they face.

\textsuperscript{13}Graf et al. (2020) show that market participants in the Italian market face lucrative outside options by participating in the re-dispatch market. We do not explicitly consider this aspect in this analysis but limit its impact by focusing on the units that optimize their offers to mainly participate in the day-ahead market.

\textsuperscript{14}In the case of the incumbent’s 3.6 GW capacity plant “Centrale termoelettrica Alessandro Volta”, which is split up into 4 units, whereas unit 1 and 2 have a 40\% share each, we decided to include unit 1 but exclude unit 2. Units 3 and 4 have a share about 10\% during the sample period and are thus not included.
This logic implies that our best-reply offer price optimization procedure finds the offer prices associated with the quantity increments of each step function offer curve that maximizes the firm’s expected profits given the distribution of residual demand surfaces faced and locational pricing market-clearing mechanism. The approach allows the supplier to withhold capacity by setting a high offer price. Endogenizing the quantity decision for each bid step could increase the firm’s expected profit, but it would also increase the computational complexity of the problem tremendously.

5.1 Cost of Electricity

We calculate the marginal cost of production for each thermal generation unit using data on its thermal efficiency and monthly prices for fuel and CO$_2$ certificates. The firm’s aggregate marginal cost function is a non-decreasing step function with the quantity step equal to the available capacity of the unit and height equal to the marginal cost of the generation unit. For thermal plants that are not optimizing the offer prices of their units, we replace the marginal cost estimate by the actual offer price in the firm’s aggregate marginal cost function.

Table 5 shows that the incumbent sets the price with pumped-hydro storage plants in 22% of the cases. Storage is a peak technology and usually assumed to offer based on an opportunity cost logic. Hence, the correct way of modeling storage plants would be to endogenize their offer behavior based on expected future electricity prices (see e.g., Graf and Wozabal, 2013). Planning a hydro storage dispatch relies on the solution to a stochastic dynamic optimization problem taking into account expected future electricity prices, as well as reservoir levels, inflows, production capacity, and consumption capacity.

Our strategy to deal with the hydro units, many of which were storage plants, is to fix their realized production for each residual demand surface scenario that the value obtained from using the unit’s actual offer curve. Specifically, we employ a two-step approach where we first solve the market-clearing for each scenario using the actual offer curves of the hydro
units. We then fix the hydro unit’s level of dispatch in every scenario by replacing the corresponding offer price with zero if a unit got dispatched and replacing it with the offer price cap otherwise. In other words, this method fixes the schedule of hydro unit for each residual demand surface scenario. This approach allows us to solve the best-response offer model in a second stage for a firm’s thermal units, conditional on the dispatch of its hydro units for each residual demand surface realization. As pointed out in Bushnell (2003), hydro storage can be used to enhance market power of a firm owning thermal units as well as hydro storage. Our approach of fixing the production of hydro units for each residual demand surface scenario implicitly accounts for the interaction between thermal units and storage units,

5.2 Eliciting Financial Forward Contract Positions

The aggressiveness of the best-reply offer curve depend on the suppliers fixed-price forward market obligations. In the Italian context, we differentiate between physically scheduled quantities of bilateral contracts and financial forward positions. Although self-schedules are identifiable from the offer data,\textsuperscript{15} financial forward positions are confidential and only known to the firm. However, the way a firm is offering its production capacity to the day-ahead market reveals some insight on their levels of forward contracts. As discussed in Wolak (2000) it would be irrational to offer capacity below marginal cost if there were no hedge contracts involved. Hence, we set $QC_i$ equal to the value that solves $\sum_z S_{iz} \left( C_{iz}(QC_{iz}) \right) = QC_{iz}$, which is the intersection between the marginal cost curve and the offer curve in each zone.\textsuperscript{16} We only utilize offer curves from generation units owned by the firm that are located in zone $z$ when solving for $QC_{iz}$.

The incumbent [main competitor] supplies on average 11.6 [6.9] GWh to the day-ahead

\textsuperscript{15}The offer data includes a unit label as well as an operator label. In case of a self-scheduled bilateral contract the operator label shows the entry “bilateral.” However, each offer is still tied to a unit operated by a firm which allows us to map all units to operators.

\textsuperscript{16}We smooth the zonal offer curve as well as the zonal marginal cost curve of thermal units in order to find their intersection. We then add the hydro production offered below 50 EUR/MWh to this value.
market in each hour. Our estimate of the hedged quantity amounts to 9.1 [5.9] GWh on average. This means that about 80% [86%] of the incumbent’s [main competitor’s] supply is hedged.

5.3 Scenario Selection

Each firm maximizes expected profits conditional on its beliefs about the distribution of the residual demand surfaces that it faces. Selecting a reasonable residual demand surface distribution is critical because it determines the uncertainty in variable profit realizations faced by the firm given its offer curve. We decided to select scenarios based on “like-markets”.

The day-ahead market clears based only the hourly offer curves and hourly demand curves submitted. Therefore, every hour of each day in our sample can be treated as a single market. We treat weekdays alike and pool hours 10, 11, and 12, and hours 18, 19, 20, and 21. We then select “like-markets” from these groups to obtain the residual demand surface realizations used to derive the best-reply offer curve. We cap the number of “like-markets” at 20, including the actual market. Note that the share of electricity generated from renewable sources was negligible in 2007, so we do not account for renewable energy production when selecting “like-markets.”

There are a few cases with supply shortage on the (energy) islands. This means that demand would exceed supply in some scenarios. We deal with this problem by adding artificial offers at a price of 300 EUR/MWh.\footnote{During this time period, the grid operator would intervene by issuing demand bids, setting a high market-clearing price if these system conditions occurred.}

The aggregated level of imports and exports coming from the explicit transmission right auction as well as the transmission capacity constraints are known before the decision how to offer at the day-ahead market is made. We therefore selected a time period with stable imports as well as similar transmission system conditions.
5.4 Best-Response Offer Curve

We solve for the best-response offer curve using the realized profit function in Equation 5. A detailed technical description of the reformulation of the model into a mixed integer program can be found in Appendix F. We solve the model by using an off the shelf mixed integer solver. We account for potential offer price and quantity indeterminacy by using the merit order number set by the system operator to guarantee uniqueness of each market-clearing outcome. The merit order number orders units in case of a tie, i.e., when their submitted offer prices are equal.\footnote{See Appendix C.3 for a more detailed description on this rule.}

An important factor in being able to solve a mixed integer program (MIP) of the size of our problem is having good starting values that can be used to warm-start the algorithm. We therefore presolve the model with marginal cost offer curves and with actual offer curves, and supply the solution to these problems as starting values to the MIP solver. Remember that for an exogenous offer curve the market-clearing problem is a linear program and hence fast to solve. Furthermore, we also supply the solution of the best-response offer problem applying the derivative free “Nelder-Mead” algorithm to the MIP solver. We aggregate bids of the competitors by price ticks in order to reduce the size of the problem. Note that any MIP is an NP-hard (non-deterministic polynomial time-hard) optimization problem which implies that an algorithm that is solvable in polynomial time might not exist.

The advantage of the mixed integer problem formulation is that we are not required to smooth the problem or make any assumptions about the functional form of the residual demand. This allows us to model the problem as it really appears to firms submitting offer curves to the day-ahead market. Furthermore, we are able to assess the quality of the solution indicated by the optimality gap (MIP gap) which is the difference in the objective function value between the current best integer solution and the optimal value of a linear program (LP) relaxation. If the gap is zero the resulting solution is the proven global optimum.

The optimal best-response offer curve may not be unique for a couple of reasons. First,
optimal offer prices are undefined for extra-marginal units. For example, if it is optimal to financially withhold capacity any offer price larger than the maximum zonal price and smaller than the price cap is optimal. Second, even for marginal units there can be several combinations of optimal offer prices that lead to the same expected profits. This can be the case, if there are large gaps between the scenarios where the optimal offer curve intersects with the inverse residual demand hyper-surfaces. Moreover, it could be theoretically possible that a best-reply offer curve that will congest the network gives the same expected profit as a best-reply offer curve that does not congest the market. However, because we fix offer quantities at their actual values, us an ample set of like-markets to calculate expected profits, and require offer prices to be greater than or equal to the unit’s marginal cost, we decrease the risk of calculating non-unique solutions.

All our computational models are implemented in Python 3.6. We use Gurobi 8.01 as a solver for the linear programs as well as the mixed integer programs. We run the mixed integer programs using four cores for each market instance with a time limit of 20,000 seconds.

6 Results

In this section, we characterize market outcome using the expected profit-maximizing offer curves. In Figures 13 and 14, we show these optimal zonal offer curves (green) for the incumbent as well as the main competitor. We furthermore, show the actual offer curves (orange), marginal cost curves (blue), and the optimized offer curves under the assumption that the firm does not take transmission constraints into account when computing their expected profit-maximizing offer curve (red). In both cases we can see that the optimal offer curve when accounting for transmission constraints looks different from the optimal offer curve when ignoring transmission constraints. In the case of the main competitor, depicted in Figure 14, we observe that the optimal offer curve in the Center zone (Panel b) would be equal to its marginal cost. This happens when the distribution of residual demand is
indefinitely elastic and the best-reply offer curve is the firm’s marginal cost function. The conclusion from these figures is that optimal zonal offer curves may look very different than the optimal offer curves assuming no transmission constraints.

In order to show the importance of the transmission network configuration on offer behavior over a wide range of system conditions, we have optimized over 180 market instances using 20 residual demand surface realizations for each instance. Table 1 shows average zonal prices as well as the average uniform purchase price and average profits for the incumbent firm and its main competitor. Column (1) shows the result of the market clearing when offer prices of the relevant units are replaced by our marginal cost estimate. Column (2) shows the market clearing results using the actual data. In column (3) we show the results of the optimal best-response offer curve accounting for transmission constraints while in column (4) we present the results of the best-response offer curves assuming infinite transmission capacity between domestic zones. In Panel A, we present the average prices, average awarded quantities of the relevant units, as well as the average profit for the incumbent. The same set of variables is presented for the main competitor in Panel B.

From column (1) and (2), we learn that offering at marginal cost would lead to a lower average profit in comparison to the average profit evaluated at actual bids. In case of the incumbent the difference is substantial. The average profit using optimal best-response offer curves (column 3) is about 4% [8%] larger for the incumbent [main competitor] compared to the average profit using actual offer curves (column 2). In column (4), we show the result of the market-clearing assuming that the firm does not account for transmission constraints when optimizing its offer curves. The difference between the optimal average profit when ignoring transmission constraints to construct the optimal offer curve (column 4 minus column 2) is 85% for the incumbent [57% for the main competitor] of the difference of the optimal average profit when accounting for the impact of transmission constraints to construct the optimal offer curve and the actual profit (column 3 minus column 2). Consequently, only a fraction of profits that could be possible when accounting for transmission constraints in com-
puting best response offer curves would be achieved if transmission constraints were ignored in the firm’s optimization procedure. This emphasizes the important role of considering the transmission network when offering to the day-ahead market.

According to a paired sample z-test on the differences between the uniform purchase price, aggregated quantities, as well as profits between columns (3) and (4) in Table 1, we reject the null that the average pair differences is not significantly different then zero at the five percent level. In other words, the clearing results when using best-response offer curves as well as accounting for transmission constraints to construct optimal differs from the clearing results when using best-response offer curves not accounting for transmission constraints.

If the incumbent were to bid its marginal cost instead of its actual bids, average market-clearing prices would be considerably lower so would the firm’s profit. Furthermore, the price difference between the North zone and the Center zone would be minimal in comparison to the actual situation. This shows the importance of the unilateral ability of the firm to cause congestion through its offer behavior. The situation appears to be different from the case of the main competitor with the ability to exercise unilateral market power only in the North and in Sicily. Prices would still be lower if the firm would submit a marginal cost offer curve instead of its actual offer curve, but by no means as extreme as for the incumbent. We furthermore observe, that the average price difference between the North and the Center would be larger if the main competitor would offer closer to its marginal cost curve. This highlights the fact that the location of a firm’s capacity together with residual demand conditions determine the level of congestion as well as zonal prices. We explore this issue in more detail in Section 7.

[Figure 13 about here.]

[Figure 14 about here.]
Table 1: Best-Response Offer Results

<table>
<thead>
<tr>
<th></th>
<th>(1) $J({c_1, g_1})$</th>
<th>(2) $J(\theta_i)$</th>
<th>(3) $J(\theta_i^*(f))$</th>
<th>(4) $J(\theta_i^*(f_\infty))$</th>
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Panel A. Incumbent

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<td>255,581</td>
</tr>
</tbody>
</table>

$J(\cdot)$ denotes the market clearing parametric on a firm’s offer curve. Col (1): Firm’s offer prices of relevant units replaced by their marginal cost estimate. Col (2): Firm’s offer curve as observed in the data. Col (3): Firm’s optimal offer curve. Col (4): Firm’s optimal offer curve assuming that transmission constraints are absent. Small market price differences in Col (2) are due to fixing production from hydro units. Quantities in MW and only of relevant thermal units, prices in EUR/MWh. All values are averages.

1 Quality of solutions supplied to $J(\cdot)$ in terms of MIP gap: Col (3): Average: 0.8%; [P₂₅: 0%, P₅₀: 0.2%, P₇₅: 0.1%, P₉₉: 5.7%]. Col (4): Average: 0.5%; [P₂₅: 0%, P₅₀: 0.2%, P₇₅: 0.7%, P₉₉: 3.3%].

2 Quality of solutions supplied to $J(\cdot)$ in terms of MIP gap: Col (3): Average: 0.2%; [P₂₅: 0%, P₅₀: 0%, P₇₅: 0.1%, P₉₉: 2.83%]. Col (4): Average: 0.1%; [P₂₅: 0%, P₅₀: 0%, P₇₅: 0%, P₉₉: 1.6%].

3 Forward market price is set to 78 EUR/MWh which is the average uniform purchase price in the sample period. Cost/revenue from importing and exporting is not considered.
7 Capacity Location and Unilateral Market Power

In this section, we analyze two counterfactual situations to highlight the importance of a generation unit owner’s location in the transmission network. We do this for the incumbent as it is the only dominant player with significant capacity in all zones. First, we explore how the results of the market clearing mechanism using the incumbent’s optimal offer curves would change as a result of a capacity relocation within the network. Second, we analyze how a capacity divestment in different zones would change the incumbent’s optimal best-reply offer curves.

We find that the optimal offer curves after transferring capacity geographically would lead to a significantly different dispatch for the system suggesting that the configuration of the transmission network plays a crucial role when suppliers construct their offers into the day-ahead market. We demonstrate that where a given megawatts (MWs) of generation capacity divestment takes places determines the extent of unilateral market power the divesting supplier is able to exercise.

7.1 Zonal Capacity Relocation

As discussed previously, the residual demand hyper-surfaces a firm faces determine its ability to exercise unilateral market power. Whether these surfaces would be affected by a capacity relocation depends on the form of the residual demand surfaces and the amount of capacity that is relocated. To make this clear, consider a two zone network with a firm facing elastic residual demand surfaces in both zones at output levels close to the firm’s maximum zonal capacity. In Figure 5, Panels (a) and (b), we observe that for a significant part of the firm’s capacity in both zones, the residual demand surfaces are essentially perfectly elastic close to the maximum capacity. Hence, transferring only a small amount of capacity from one zone

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19 Remember, in a single-pricing-market the location of capacity does not play a role and offering will be based on the merit ordering of the firm’s capacities. That is because the competitive conditions a supplier faces only depends on the offers of its competitors, regardless of where they are located in the transmission network.
to the other would not impact the residual demand surface for a wide range of quantities. More generally, the firm’s ability to assess market power will only be affected if the capacity relocation affects the part of the residual demand hyper-surface where the firm is pivotal or faces a steep residual demand curve.

The effect on the residual demand hyper-surface is, however, not the only effect a capacity relocation has. The other effect is the cost-structure effect. Any relocation of capacity leads to a change in the zonal cost structure of a firm. Depending on which part of the firm’s aggregate marginal cost function is affected by the capacity transfer this effect may impact the way a firm is offering to the market.

In order to demonstrate the predictive power of our model, we deploy two potential capacity transfers summarized in Table 2. First, we move the incumbent’s coal capacity in the Center to the North. The incumbent exerts market power with part of its coal capacity in the Center. The results are depicted in Column (2). In Column (3) [Column (4)] we show the absolute [relative] difference between the status quo, Column (1), and the counter-factual. We find that having cheaper capacity available in the North would lead to an increase in the average dispatch in the North by 765 MWh relative to the status quo under our expected profit-maximizing offer solution with this locational configuration of capacity. This would lead to a zonal price decrease of 12%. At the same time the relative dispatch in the center would decrease by 648 MWh on average, resulting in an average zonal price increase of 5%. Overall the firm would offer more aggressively than it does in the status quo, leading to 8% more dispatched quantity and a reduced uniform purchase price of 4%. The difference in the uniform purchase price and in the aggregate quantity between the actual values of zonal capacity and the counterfactual value of zonal capacity is statistically different from zero at the one percent level according to a paired z-test.

The second capacity transfer is to move the incumbent’s relevant coal capacity in the North to the Center. Results are presented in Columns (5)–(7). We find that the average dispatch would decrease by 170 MWh in the North but would increase only by 139 MWh on
average in the Center. Average prices hardly change. Although, the difference in aggregate quantity is significantly different from zero, the difference in the uniform purchase price is not statistically different from zero at the five percent level.

One conclusion of this exercise is that the incumbent firm has cheap (coal) capacity in a zone with an inelastic residual demand as depicted in Fig 7, Panel (b). As a result of this it is profitable for the firm to withhold part of this capacity from the market. If the firm’s coal capacity were to be moved from the Center to the North, it would increase its overall production by 8%. On the other hand if the firm’s coal capacity in the North were moved to the Center it it would even produce slightly less than before. From this exercise, we conclude that pattern of congestion depends the firm’s zonal distribution of capacity and not only on demand conditions. This becomes evident as we move the coal capacity from the Center to the North with the result that the average price difference between the North and the Center decreases by 71%.

7.2 Zonal Capacity Divestment

As a final exercise, we demonstrate that the location of a capacity divestment in the transmission network affect the incumbent’s best-response offer curve and resulting market-clearing zonal prices. We focus again on the incumbent as it has a substantial amount of thermal capacity in all zones. We deploy four scenarios, two where we divest about 1.2 GW of the incumbent’s thermal capacity in the North and in the Center, respectively. In the remaining two scenarios, we divest about 2.4 GW in each of the two zones. In order to make the counterfactuals comparable to the status quo, we adjust the firm’s forward commitments by subtracting the quantity of the divested plants that was offered below marginal cost. We also assume that the capacity is divested to a fringe player, meaning that the divested capacity is offered at marginal cost to the market. In order to make this exercise as realistic as possible, we decided to divest whole units. The actual as well as the optimal offer strategy of the incumbent suggests that differentiation between offer prices of relevant coal and gas units is
Table 2: Fictive Transfers of Incumbent’s Capacity

<table>
<thead>
<tr>
<th></th>
<th>(1) $\bar{p}$</th>
<th>(2) $p_{NC}$</th>
<th>(3) $p_{SI}$</th>
<th>(4) $q_{t}$</th>
<th>(5) $q_{t,N}$</th>
<th>(6) $q_{t,C}$</th>
<th>(7) $q_{t,SI}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N$</td>
<td>$J(\theta^*_1)$</td>
<td>$J(\theta^*_1)$</td>
<td>$\Delta$</td>
<td>$\Delta(%)$</td>
<td>$J(\theta^*_1)$</td>
<td>$\Delta$</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>180</td>
<td>105.10</td>
<td>101.05</td>
<td>-4.04</td>
<td>-4%</td>
<td>104.84</td>
<td>-0.26</td>
</tr>
<tr>
<td>$p_{NC}$</td>
<td>180</td>
<td>92.44</td>
<td>81.15</td>
<td>-11.30</td>
<td>-12%</td>
<td>93.00</td>
<td>0.56</td>
</tr>
<tr>
<td>$p_{SI}$</td>
<td>180</td>
<td>116.98</td>
<td>123.18</td>
<td>6.20</td>
<td>5%</td>
<td>115.34</td>
<td>-1.64</td>
</tr>
<tr>
<td>$q_{t}$</td>
<td>180</td>
<td>153.51</td>
<td>154.50</td>
<td>0.98</td>
<td>1%</td>
<td>154.31</td>
<td>0.80</td>
</tr>
<tr>
<td>$q_{t,N}$</td>
<td>180</td>
<td>1,821</td>
<td>1,967</td>
<td>146</td>
<td>8%</td>
<td>1,777</td>
<td>-45</td>
</tr>
<tr>
<td>$q_{t,C}$</td>
<td>180</td>
<td>272</td>
<td>1,037</td>
<td>765</td>
<td>281%</td>
<td>102</td>
<td>-170</td>
</tr>
<tr>
<td>$q_{t,SI}$</td>
<td>180</td>
<td>1,344</td>
<td>697</td>
<td>-648</td>
<td>-48%</td>
<td>1,483</td>
<td>139</td>
</tr>
</tbody>
</table>

Col (1): Average results of market clearing evaluated at incumbent’s optimal offer curve. Col (2): Average results of market clearing evaluated at market clearing with incumbent’s optimal offer curve assuming that coal capacity of relevant units were transferred from the Center to the North. Col (3): Actual change between Col (1) and Col (2). Col (4): Relative change between Col (1) and Col (2). Col (5): Average results of market clearing evaluated at market clearing with incumbent’s optimal offer curve assuming that coal capacity of relevant units were transferred from the North to the Center. Col (6): Actual change between Col (1) and Col (5). Col (7): Relative change between Col (1) and Col (5). Quantities in MW and only of relevant thermal units, prices in EUR/MWh.

1 Quality of solutions supplied to the market clearing algorithm in terms of MIP gap: Average: 0.8%; [P_{25}: 0%, P_{50}: 0.2%, P_{75}: 0.1%, P_{90}: 5.7%].
2 Quality of solutions supplied to the market clearing algorithm in terms of MIP gap: Average: 0.8%; [P_{25}: 0%, P_{50}: 0.3%, P_{75}: 1.0%, P_{90}: 5.2%].
3 Quality of solutions supplied to the market clearing algorithm in terms of MIP gap: Average: 0.8%; [P_{25}: 0%, P_{50}: 0.2%, P_{75}: 1.0%, P_{90}: 5.2%].

not optimal (see Figure 13), hence, we select mainly coal plants.20

In Table 3, we present the results. A divestment of the incumbent’s thermal capacity leads to lower best-response prices as expected. However, the zonal configuration of the market turns out to be an important aspect when comparing our different divestment scenarios. Divesting 1.2 GWs of the incumbent’s thermal capacity in the North or in the Center (Panel A) leads to a similar reduction in the uniform purchase price relative to the status quo (compare columns (2) and (5) to column (1) in Table 3). Although, the channels leading to this result are slightly different, a divestment in the North would lead to a lower average

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20In the North, we divested units 1 to 4 of the Fusina plant and unit 3 of the Spezia plant. For the large divestment we added units 3, 4, and 6 of the Genoa plant and units 3 and 4 of the Porto Corsini plant. In the Center we divested units 1 and 2 of the Brindisi plant, and for the large divestment we added units 3 and 4 of the Brindisi plant.
zonal price in this zone and a slightly higher price in the Center. Hence, the incumbent’s unilateral incentive to drive prices in the two zones apart would increase. However, since the North zone is larger than the Center zone, the decrease of the zonal price in the North dominates the slight increase in the Center and hence, the best-response uniform purchase price would be lower. When divesting in the Center, both zonal prices would decrease, and so would the best-response uniform purchase price. Paired z-tests where we compare the best-response uniform purchase price in the status quo with the uniform purchase price when divesting in the Center and in the North reveal that the means are indeed different. On the other hand, the paired z-test confirms that means of the uniform purchase prices are statistically equal independent of divesting in the Center or in the North.

The zonal configuration of the market becomes even more important when applying a large divestment counterfactual scenario (2.4 GW). In this case the location of where to divest in the network, may severely impact the uniform purchase price. We provide the results of this exercise in Panel B of Table 3. Divesting a large amount of the incumbent’s thermal capacity in the North does not lead to a significant change of the best-response uniform purchase price in comparison to the small divestment (compare row $\bar{p}$, column (5) in Panel A to the same row and column in Panel B in Table 3). The large capacity divestment leads even to a slightly larger best-response uniform purchase price. However, a paired z-test does not reject the null that the expected value of the difference between them will be zero. The reasons for this is that such a divestment strategy would lead to an increase in the best-response price in the Center which does not compensate the decrease of the best-response price in the North.

We get the most significant effect by applying a large divestment of the incumbent’s thermal capacity in the Center. As a result of such a divestment, the best-response price in the Center would drop by 14% relative to the status quo. This has a severe effect on the uniform purchase price which will drop by 7%. Paired z-tests where we compare the best-response uniform purchase price in the status quo with the uniform purchase price when
divesting a large amount of capacity in the Center and in the North reveal that the means are indeed different. This demonstrates that the same number of MWs of divestment in different zones would yield different counterfactual uniform purchase prices, highlighting the importance of a locational perspective when making a divestment decision.

The finding that capacity location may be critical when measuring market power is in line with Bigerna et al. (2016) who assess the ability of suppliers to exercise unilateral market power in the Italian day-ahead market using zonal Lerner indices. Their results show that generators are only able raise prices above competitive levels in specific congestion zones.

The conclusion of this exercise is that locational aspects may be critical for evaluating divestment strategies. Our tool is flexible in a sense that any modification of a firm’s capacity structure is possible. As such, it may be an important tool for companies and regulators alike. For the former to study the expected profitability of a technology upgrade or to study the expected profitability of an acquisition of a new plant. For the regulator, the tool may be useful to study which zonal divestment strategy would make the market more competitive.

8 The Effect of a Transmission Configuration Change

The best-reply offer curve method described and applied in this paper can also be used to evaluate a transmission configuration change. This includes an upgrade of the transmission capacity between two zones or a different zonal configuration at all.

Specifically, one could solve for the best-reply offer curves of large suppliers with and without the configuration change and compare the cost of wholesale electricity to consumers before and after the change. To capture the full effect of a transmission configuration change, one should also take into account the interaction among strategic players. One attempt to achieve that could be to formulate the problem as a stochastic Equilibrium Problem with Equilibrium Constraints (EPEC) in which each firm solves a stochastic MPEC. Yao et al. (2008); Ito and Reguant (2016) use an iterative solution method to solve an EPEC for simple
### Table 3: Divestment of Incumbent’s Capacity

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Status quo</td>
<td>Divestment in the Center</td>
<td>Divestment in the North</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>$J(\theta^*_i)$</td>
<td>$\tilde{J}(\theta^*_i)$</td>
<td>$\Delta$</td>
<td>$\Delta(%)$</td>
<td>$\tilde{J}(\theta^*_i)$</td>
<td>$\Delta$</td>
<td>$\Delta(%)$</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>180</td>
<td>105.10</td>
<td>103.27</td>
<td>-1.82</td>
<td>-2%</td>
<td>103.59</td>
<td>-1.51</td>
</tr>
<tr>
<td>$p_N$</td>
<td>180</td>
<td>92.44</td>
<td>91.83</td>
<td>-0.61</td>
<td>-1%</td>
<td>88.99</td>
<td>-3.45</td>
</tr>
<tr>
<td>$p_C$</td>
<td>180</td>
<td>116.98</td>
<td>112.94</td>
<td>-4.04</td>
<td>-3%</td>
<td>117.83</td>
<td>0.85</td>
</tr>
<tr>
<td>$p_{SI}$</td>
<td>180</td>
<td>153.51</td>
<td>154.16</td>
<td>0.64</td>
<td>0%</td>
<td>155.91</td>
<td>2.40</td>
</tr>
</tbody>
</table>

**Panel A. Thermal capacity divestment of about 1.2 GW**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Status quo</td>
<td>Divestment in the Center</td>
<td>Divestment in the North</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>$J(\theta^*_i)$</td>
<td>$\tilde{J}(\theta^*_i)$</td>
<td>$\Delta$</td>
<td>$\Delta(%)$</td>
<td>$\tilde{J}(\theta^*_i)$</td>
<td>$\Delta$</td>
<td>$\Delta(%)$</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>180</td>
<td>105.10</td>
<td>98.06</td>
<td>-7.04</td>
<td>-7%</td>
<td>103.69</td>
<td>-1.40</td>
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<td>-4.62</td>
</tr>
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<td>$p_C$</td>
<td>180</td>
<td>116.98</td>
<td>101.09</td>
<td>-15.89</td>
<td>-14%</td>
<td>119.60</td>
<td>2.62</td>
</tr>
<tr>
<td>$p_{SI}$</td>
<td>180</td>
<td>153.51</td>
<td>153.91</td>
<td>0.39</td>
<td>0%</td>
<td>157.63</td>
<td>4.12</td>
</tr>
</tbody>
</table>

Col (1): Average results of market clearing $J(\cdot)$ evaluated at incumbent’s optimal offer curve. Col (2): Average result of market clearing $\tilde{J}(\cdot)$ evaluated at incumbent’s optimal offer curve assuming that a coal unit in the Center was divested. Col (3): Actual change between Col (1) and Col(2). Col (4): Relative change between Col (1) and Col(2). Col (5): Average results of market clearing $\tilde{J}(\cdot)$ evaluated at incumbent’s optimal offer curve assuming that a coal unit in the North was divested. Col (6): Actual change between Col (1) and Col(5). Col (7): Relative change between Col (1) and Col(5). Prices in EUR/MWh.

1 Quality of solutions supplied to the market clearing algorithm in terms of MIP gap: Col (1): Average: 0.8%; [P25: 0%, P50: 0.2%, P75: 0.1%, P99: 5.7%]. Col (2): Average: 1.5%; [P25: 0%, P50: 0.1%, P75: 2.2%, P99: 9.0%]. Col (5): Average: 0.5%; [P25: 0%, P50: 0%, P75: 0.5%, P99: 4.7%].

2 Quality of solutions supplied to the market clearing algorithm in terms of MIP gap: Col (1): Average: 0.8%; [P25: 0%, P50: 0.2%, P75: 0.1%, P99: 5.7%]. Col (2): Average: 1.4%; [P25: 0%, P50: 0.1%, P75: 1.0%, P99: 13.8%]. Col (5): Average: 0.3%; [P25: 0%, P50: 0%, P75: 0.2%, P99: 2.7%].

linear quadratic Cournot models. Unfortunately, the best-response bidding problem with endogenous congestion is already computationally challenging to solve and little is known about the quality of the solution resulting from the iterative approach to solve the EPEC problem. Hence, we leave this exercise to future research. The advantage of our approach is that we make use of the actual mode of competition, i.e., in offer curves. Furthermore, we also apply the market clearing as it appears to be in reality including the actual underlying network model that is used to set prices. This differentiates us from other papers, that use a Cournot model that ignores both actual form of competition and the network model. Furthermore, as pointed out in Borenstein et al. (2000), the Cournot model with endogenous congestion does not necessarily yield a unique equilibrium, not even in its simplest version,
i.e., with linear residual demand curve and quadratic cost curve.

Formulating and solving the best-reply offer curve model as a mixed integer program allows us to solve the problem to global optimality in many cases and gives us a metric on how good the obtained solution is compared to the best objective value an integer solution could potentially have for all cases (MIP gap).

9 Discussion and Conclusion

We have introduced the notion of residual demand hyper-surfaces, which is the natural extension of the residual demand curve concept developed for single-zone markets to multiple-zone markets connected by finite transmission capacity. These hyper-surfaces make it possible to infer the effect of a firm’s zonal supply on the zonal price in this zone and all other zones. Residual demand hyper-surfaces provide measures of the ability of a supplier to exercise unilateral market power in each pricing zone.

We have adapted the best-response offer curve model to incorporate these transmission-constrained residual demand hyper-surfaces and have implemented for the Italian wholesale electricity market. We have found that taking transmission constraints into account helps to explain actual offer behavior significantly better than a price-setting process that does not incorporate the impact of transmission network constraints.

We have studied the importance of the transmission network configuration in determining expected profit-maximizing offer curves through several counterfactual analysis. We have shown that a transfer of the incumbent’s capacity to different locations in the transmission network would lead to significantly different offer behavior and market-clearing prices and quantities. In another set of counterfactual scenarios, we have documented the importance of taking the location of a firm’s capacity into account when assessing the impact of divestment scenarios to enhance market competitiveness. The latter analysis may be very useful to regulators who may want to anticipate how a certain divestment strategy would change a
firm’s offer behavior and market-clearing prices and quantity.
References


A Introduction

The appendix is organized as follows. In Section B, we give a detailed description of the Italian electricity market. We then describe the zonal market-clearing engine deployed by the market operator in Section C. In Section D, we present a general algorithm for computing residual demand hyper-surfaces for an arbitrary locational pricing market. In Section E, we discuss some counter-intuitive implications for residual demand curves that arise when neglecting existing transmission constraints. Finally, in Section F, we show how to reformulate the best-response offer problem into a mixed integer program and solve for best-response offer curves in a locational pricing market.

B Market Description

B.1 The Italian Electricity Market

The Italian spot market for electricity began operation in April 2004. It is organized in a sequential manner, similar to the Spanish power market described in Ito and Reguant (2016). More precisely, it consists of a day-ahead market, an adjustment market,\(^{21}\) and an ancillary services market, also known as re-dispatch market. After the day-ahead market has cleared, market participants alter their day-ahead schedules in the adjustment market. Eventually, Terna, the transmission network operator, builds up reserves and resolves congestion in the so-called ancillary services or re-dispatch market.

The Italian day-ahead market for electricity is a non-compulsory net-pool zonal market administrated by the market operator. The market splits into several market zones when congestion is present. In such a case producers receive a different price depending on the zones where they are producing. Demand side, however, is paid a uniform purchase price

\(^{21}\)The adjustment market has been replaced by a series of intra-day markets in later years. The Italian Power Exchange provides a detailed description on their website, see http://www.mercatoelettrico.org/En/Mercati/MercatoElettrico/MPE.aspx.
(UPP). The day-ahead market is a so-called “energy-only” market where participants bid an offer/demand curve. Suppliers submit energy offer curves that do not include a start-up or minimum load cost. Offer and supply bid prices are floored at zero and capped at 500 EUR which was lifted to 3,000 EUR in 2008. In 2007 the maximum observed UPP was 242.4 EUR/MWh (Bosco et al., 2013). For each production or consumption unit a maximum of four bid steps is allowed. Electricity is traded for the 24 hours of the next day and there are no inter-temporal constraints, such as block bids, to consider when clearing the market. This allows us to treat each hour of the day independent from each other.

Many market participants have bilateral long-term contract obligations to supply and purchase electricity. In the day-ahead market, participants may self-schedule (a part of) their bilateral commitments. Self-scheduled demand and supply enter the day-ahead market as price inelastic bids.\(^\text{22}\) Following the definition in Anderson et al. (2007), the day-ahead market can therefore be described as a “net-pool”. In case of transmission congestion, an explicit transmission capacity fee (CCT)\(^\text{23}\) is due for the self-schedules. The CCT is defined as the difference between the hourly purchase price in the withdrawal zones of the contract and the hourly electricity selling price in the injection zones of the contract. Hence, the CCT translates into a cost for injection into exporting zones, as it contributes to increasing congestion; and it translates into a subsidy for injection into importing zones, as it contributes to relieving congestion. The CCT is zero if no congestion is present. Note that regular day-ahead market transactions pay/receive an implicit congestion fees based on the zonal price difference (for further details see Ardian et al., 2018).\(^\text{24}\)

In 2007, the Italian market consisted of seven domestic zones, five limited production zones and six foreign virtual zones to organize imports and exports. It is a radial network. Congestion between zones mostly took place between the North zone and the rest of the

\(^{22}\)There is the theoretical possibility to self-schedule demand or supply at a positive price but it is hardly used by market participants.


\(^{24}\)For more details on the market we refer to http://www.mercatoelettrico.org/En/MenuBiblioteca/documenti/20111216Annual_Report_2010.pdf.
mainland as well as between the mainland and the islands. During peak hours of our sample period, the price difference between the North zone and the rest of the mainland was about 21 EUR/MWh in case of congestion, and it occurred in over 60% of the time. We refer to Appendices B.4–B.6 for more details on the transmission network and the prevalence of congestion.

B.2 Market Structure

In 2007, electricity demand reached about 340 TWh net of energy for pumping and before transmission line losses. Peak demand was in December with about 56.8 GW. Available net installed capacity was 77.6 GW. The demand in the day-ahead market reached 330 TWh, whereas about 2/3 of the demand was fulfilled through market transactions and the remainder through self-schedules.

All customers have the right to choose their electricity supplier since July 2007. However, most domestic consumers have not opted for this option. For those households and small companies, the single buyer—a state controlled entity—is in charge for procuring electricity from the day-ahead market and reselling it to retailers (ARERA, 2008a).

In 2007, 85% of gross electricity production came from thermal plants and 13% from hydro (including some pumped-hydro storage units). The remaining share came from geothermal sources and wind power. In terms of thermal capacity, the majority is gas followed by coal and oil (see e.g., Graf and Marcantonini, 2017, for more details). Compared to Italy’s neighbors in the North— with nuclear power as the dominant source in France and hydro power in Switzerland—this is a rather expensive power production mix which explains the large amount of imports.

The incumbent firm in the Italian market is Enel—the former state-owned monopolist. Its net share of electricity generations was about 31% in 2007. However, the net available capacity (available at least 50% of the time) was 48.2% (ARERA, 2008a).

Enel’s main competitor at the time was Edison. It was controlled by Aem—a Milan
based utility and Edf—the French state-owned company. Furthermore, Edison controlled half of Edipower’s capacity via a tolling agreement. Edipower is a generation company with capacity in the North zone as well as in Sicily. Furthermore, Aem controlled 20% of Edipower. We decided to model Edison, Aem, Edf, and the 70% share in Edipower as one company construct, which we refer to as Edison in the remainder of this paper.\textsuperscript{25} Edison’s market share in generated electricity was about 13.7% in 2007 and Edipower’s 8%. In terms of available capacity, Edison’s share was 10.6% and Edipower’s 8.2% (ARERA, 2008a).\textsuperscript{26}

Both firms operate hydro plants for other firms in the north of the country. However, in most of the cases the two firms hold also company shares of these other firms, and thus we decided to add the hydro capacity to the firms’ capacities.

Some thermal plants in the Italian market are operated by a state-entity due to a special regime called CIP-6/92.\textsuperscript{27} Operatively, the state entity signed contracts for differences with the owners of these thermal plants and in exchange offered the capacity at zero to the market. This leaves the state-entity with the price risk of the short-term market. If reselling of contracts were forbidden, CIP-6 contracts would enter the firms’ objective function. However, only the net-position of contracted quantity matters when offering to the short-term market and therefore, we do only implicitly account for the contracts signed with the state-entity. We explain how we elicit the net contract position from a firm’s offer data in Section 5.2.

In Table 4, we show the zonal distribution by production type. The data in the table reveals that the incumbent has a very dominant position in the Center zone.\textsuperscript{28} Capacity shares of both firms are more balanced in the North zone. Market concentration is high

\begin{footnotesize}
\begin{itemize}
\item \textsuperscript{25}See the European Commission’s merger case “COMP/M.3729–EDF/AEM/EDISON” for more details on the firm construct including the tolling agreement.
\item \textsuperscript{26}Aem’s shares are not explicitly mentioned in the report.
\item \textsuperscript{27}This is a controversial resolution put in place before the market liberalization, with the purpose to subsidize the production of electricity from renewable and “assimilated” sources. In practice, many thermal plants, such as co-generation plants, managed to benefit from the CIP-6 subsidy by claiming to be “assimilated” to renewables. The electricity produced from “assimilated” sources under the CIP-6 agreement is not negligible as it comprised approximately 15% of the domestic thermoelectric production in 2007, (see, e.g., ARERA, 2008b). Note that this law is in stark contrast to EU law and was therefore phased out.
\item \textsuperscript{28}Our approach to elicit capacity data from the day-ahead offer data was to take the maximum offers per unit, day, and hour in the year 2007. This method yields a total supply capacity of 77.9 GW which is in line with the reported number of 77.6 GW available net capacity, see (ARERA, 2008a)
\end{itemize}
\end{footnotesize}
on the islands, Sardinia and Sicily. A further indication for the incumbent’s market power in the Center zone is that it always sets the price in the Center zone as can be seen from Table 5. The data displayed in this table shows only peak hours during the relevant sample period. In terms of technology, mostly thermal plants set prices followed by pumped-hydro storage plants.

Table 4: Capacity and Peak Demand

<table>
<thead>
<tr>
<th>Type/Zone</th>
<th>Panel A. Incumbent</th>
<th>Panel B. Main competitor</th>
<th>Panel C. Market</th>
<th>Panel D. Domestic peak demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N(^1)</td>
<td>C(^2)</td>
<td>SI(^1)</td>
<td>Total</td>
</tr>
<tr>
<td>Thermal</td>
<td>7,340</td>
<td>11,742</td>
<td>2,644</td>
<td>21,726</td>
</tr>
<tr>
<td>Pumped-hydro storage(^3)</td>
<td>4,579</td>
<td>2,106</td>
<td>583</td>
<td>7,268</td>
</tr>
<tr>
<td>Hydro(^4)</td>
<td>4,201</td>
<td>1,549</td>
<td>60</td>
<td>8,811</td>
</tr>
<tr>
<td>Other(^6)</td>
<td>760</td>
<td>105</td>
<td></td>
<td>866</td>
</tr>
<tr>
<td>Total</td>
<td>16,120</td>
<td>16,158</td>
<td>3,393</td>
<td>35,671</td>
</tr>
<tr>
<td></td>
<td>7,032</td>
<td>3,608</td>
<td>974</td>
<td>11,615</td>
</tr>
<tr>
<td>Pumped-hydro storage(^3)</td>
<td>303</td>
<td></td>
<td></td>
<td>303</td>
</tr>
<tr>
<td>Hydro(^4)</td>
<td>2,064</td>
<td></td>
<td></td>
<td>2,064</td>
</tr>
<tr>
<td>Other(^6)</td>
<td>141</td>
<td></td>
<td></td>
<td>141</td>
</tr>
<tr>
<td>Total</td>
<td>9,400</td>
<td>3,749</td>
<td>974</td>
<td>14,123</td>
</tr>
<tr>
<td></td>
<td>42,162</td>
<td>31,104</td>
<td>5,855</td>
<td>79,121</td>
</tr>
</tbody>
</table>

| Total                      | 30,638           | 20,797         | 3,229           | 54,664                      |

Method: Capacity values in MW are derived by aggregating maximum offered quantities in the day-ahead market in the year 2007. Values are in line with the ones reported by the firms in their annual operations reports. Domestic peak demand is derived from day-ahead market data and are inline with the numbers in the national power market report. Note that the hydro capacity is not available throughout the year which explains the gap between installed capacity and peak demand.

\(^1\) Includes capacity in limited production poles.

\(^2\) Center (C) zone includes zones CN, CS, CA, S, and SA and limited production poles therein.

\(^3\) Production capacity of pumped-hydro storage plants.

\(^4\) Includes run-off the river plants, hydro systems with dams, and reservoirs.

\(^5\) Production capacity from geothermal (686 MW in C), wind, and photovoltaics.
Table 5: Count of Marginal Unit by Firm/Type

<table>
<thead>
<tr>
<th>Type/Zone</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N\textsuperscript{1}</td>
</tr>
<tr>
<td>Panel A. Incumbent</td>
<td></td>
</tr>
<tr>
<td>Thermal</td>
<td>16</td>
</tr>
<tr>
<td>Pumped-hydro storage</td>
<td>14</td>
</tr>
<tr>
<td>Hydro\textsuperscript{3}</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>38</td>
</tr>
<tr>
<td>Panel B. Main competitor</td>
<td></td>
</tr>
<tr>
<td>Thermal</td>
<td>45</td>
</tr>
<tr>
<td>Pumped-hydro storage</td>
<td>4</td>
</tr>
<tr>
<td>Hydro\textsuperscript{3}</td>
<td>18</td>
</tr>
<tr>
<td>Total</td>
<td>67</td>
</tr>
<tr>
<td>Panel C. Market</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>110</td>
</tr>
</tbody>
</table>

Method: Offer price equals to awarded price and awarded quantity strictly smaller than submitted quantity. This approach rules out cases where marginal plants got their whole quantity awarded. Probability for such an event should be very low. Note that due to the endogenous uniform purchase price procedure it is possible that more than one plant is marginal within a price zone. Only peak hours and only units in Italian zones during the relevant period in 2007 are considered.

\textsuperscript{1} Includes capacity in limited production poles.

\textsuperscript{2} Center (C) zone includes zones CN, CS, CA, S, and SA and limited production poles therein.

\textsuperscript{3} Includes run-off the river plants, hydro systems with dams, and reservoirs.

B.3 Day-Ahead Market Bids and Transactions

Table 7 shows market bids and transaction for the sample period. A large share of domestic supply bids is price inelastic and domestic demand bids are almost entirely inelastic. Imports play a considerable role in the Italian market as 15% of electricity is imported to cover domestic purchases. The amount of pumping electricity is negligible since our sample is restricted to peak hours only.
Table 6: Average Day-Ahead Market Demand, Supply, and Prices by Congestion

<table>
<thead>
<tr>
<th></th>
<th>(1) N^1</th>
<th>(2) C^2</th>
<th>(3) SI^1</th>
<th>(4) Total</th>
<th>(5) N^1</th>
<th>(6) C^2</th>
<th>(7) SI^1</th>
<th>(8) Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic demand</td>
<td>25,093</td>
<td>16,799</td>
<td>2,462</td>
<td>44,354</td>
<td>25,594</td>
<td>17,081</td>
<td>2,570</td>
<td>45,245</td>
</tr>
<tr>
<td>Incumbent’s supply</td>
<td>6,333</td>
<td>4,097</td>
<td>1,337</td>
<td>11,767</td>
<td>6,266</td>
<td>4,473</td>
<td>1,348</td>
<td>12,088</td>
</tr>
<tr>
<td>Share</td>
<td>0.25</td>
<td>0.24</td>
<td>0.54</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.27</td>
</tr>
<tr>
<td>Main competitor’s supply</td>
<td>5,756</td>
<td>3,258</td>
<td>368</td>
<td>9,382</td>
<td>6,851</td>
<td>3,319</td>
<td>444</td>
<td>10,614</td>
</tr>
<tr>
<td>Share</td>
<td>0.23</td>
<td>0.19</td>
<td>0.15</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.23</td>
</tr>
<tr>
<td>Zonal price</td>
<td>88.19</td>
<td>123.33</td>
<td>128.02</td>
<td>–</td>
<td>120.36</td>
<td>120.36</td>
<td>132.34</td>
<td>–</td>
</tr>
<tr>
<td>Uniform purchase price</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>103.31</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>120.84</td>
</tr>
</tbody>
</table>

Method: Only peak hours during the relevant period in 2007 are considered. Average quantities in MWh and average prices in EUR/MWh. 141 out of 180 hours are constrained.

1 Constrainedness is defined as price difference between zone North and and zone South
2 Includes capacity in limited production poles.
3 Center (C) zone includes zones CN, CS, CA, S, and SA and limited production poles therein.

B.4 Transmission Network

Figure 15 shows the transmission network structure in 2007 relevant for the day-ahead market clearing. It is a radial representation where nodes are aggregated to zones. The zones can be subdivided into domestic zones, limited production zones and foreign zones. Domestic zones contain generation as well as load while limited production zones only contain generation. The Italian power system is highly interconnected with its neighboring countries in the North, especially, France (FRA) and Switzerland (CHE). Italian as well as foreign players can make sale or purchase offers of electricity in the foreign virtual zones.

[Figure 15 about here.]

29Domestic zones are: North (N), Center-North (CN), Center-South (CS), Sardinia (SA), South (S), Calabria (CA), and Sicily (SI); Limited production zones are Brindisi, Foggia, Monfalcone, Priolo Gargallo, Rossano, Turbigo-Ronco; and Foreign zones are: France (FRA) including Corsica (CO), Switzerland (CHE), Austria (AUT), Slovenia (SVN), and Greece (GRC). A detailed list of market zones can be found here: http://www.mercatoelettrico.org/en/mercati/mercatoelettrico/zone.aspx.
Table 7: Day-Ahead Market Bids and Transactions

<table>
<thead>
<tr>
<th>Variable</th>
<th>MWh</th>
<th>Share</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Bids</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domestic supply bids</td>
<td>9,635,233</td>
<td>1.00</td>
<td>199,470</td>
</tr>
<tr>
<td>Price inelastic</td>
<td>4,199,716</td>
<td>0.44</td>
<td>85,148</td>
</tr>
<tr>
<td>Physical schedules</td>
<td>2,275,140</td>
<td>0.24</td>
<td>38,812</td>
</tr>
<tr>
<td>Pumping bids</td>
<td>-560</td>
<td>1.00</td>
<td>7</td>
</tr>
<tr>
<td>Price inelastic</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>Physical schedules</td>
<td>0</td>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>Import bids</td>
<td>1,301,557</td>
<td>1.00</td>
<td>25,034</td>
</tr>
<tr>
<td>Price inelastic</td>
<td>1,134,389</td>
<td>0.87</td>
<td>21,886</td>
</tr>
<tr>
<td>Physical schedules</td>
<td>791,372</td>
<td>0.61</td>
<td>12,049</td>
</tr>
<tr>
<td>Export bids</td>
<td>258,895</td>
<td>1.00</td>
<td>2,971</td>
</tr>
<tr>
<td>Price inelastic</td>
<td>123,371</td>
<td>0.48</td>
<td>1,332</td>
</tr>
<tr>
<td>Physical schedules</td>
<td>34,420</td>
<td>0.13</td>
<td>595</td>
</tr>
<tr>
<td>Domestic demand bids</td>
<td>8,023,842</td>
<td>1.00</td>
<td>74,238</td>
</tr>
<tr>
<td>Price inelastic</td>
<td>8,018,478</td>
<td>1.00</td>
<td>73,554</td>
</tr>
<tr>
<td>Physical schedules</td>
<td>2,722,469</td>
<td>0.34</td>
<td>43,550</td>
</tr>
<tr>
<td><strong>Panel B. Transactions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domestic Sales</td>
<td>6,985,773</td>
<td>0.87</td>
<td>152,220</td>
</tr>
<tr>
<td>Pumping</td>
<td>-560</td>
<td>0.00</td>
<td>7</td>
</tr>
<tr>
<td>Imports</td>
<td>1,189,048</td>
<td>0.15</td>
<td>23,141</td>
</tr>
<tr>
<td>Exports</td>
<td>-155,783</td>
<td>-0.02</td>
<td>1,702</td>
</tr>
<tr>
<td>Domestic Purchases</td>
<td>8,018,478</td>
<td>1.00</td>
<td>73,554</td>
</tr>
</tbody>
</table>

Method: Only peak hours during the relevant period in 2007 are considered (sample data) which sum up to 180 market instances. Physical schedules are included in the price inelastic bids.

### B.5 Zonal Prices

Figure 16 shows the distributions of zonal day-ahead market prices between September 1st and November 1st, 2007. Two things are notable from this figure: (i) all zonal price distribution except in the North zone (N) show heavy tails, (ii) the uniform purchase price (UPP) does a good job in absorbing extreme prices observed in all the zones south of the North zone.

[Figure 16 about here.]
B.6 Prevalence of Congestion

Table 8 shows the zonal price differences and occurrences between September 1st and November 1st, 2007. The data in the table reveals that in about 40% of the cases, the price in the mainland zones except North (CN, CS, S, CA) is higher than in the North zone (N). This share is even higher when comparing to the prices on the two island (SA, SI). Conditional on observing a higher price in all other zones except the North zone, the price difference is about 21 EUR/MWh.

Table 8: Prevalence of Congestion

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>CN</th>
<th>CS</th>
<th>SA</th>
<th>S</th>
<th>CA</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A. Row price higher than column (Share)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>0.03</td>
<td>0.04</td>
<td>0.14</td>
<td>0.05</td>
<td>0.06</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>CN</td>
<td>0.38</td>
<td>0.03</td>
<td>0.16</td>
<td>0.03</td>
<td>0.06</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>CS</td>
<td>0.38</td>
<td>0.00</td>
<td>0.16</td>
<td>0.01</td>
<td>0.04</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>SA</td>
<td>0.36</td>
<td>0.04</td>
<td>0.06</td>
<td>0.07</td>
<td>0.08</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>0.38</td>
<td>0.00</td>
<td>0.00</td>
<td>0.16</td>
<td>0.03</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>CA</td>
<td>0.38</td>
<td>0.00</td>
<td>0.16</td>
<td>0.00</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SI</td>
<td>0.62</td>
<td>0.41</td>
<td>0.41</td>
<td>0.52</td>
<td>0.42</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B. Row price less column price, conditional on being higher

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>CN</th>
<th>CS</th>
<th>SA</th>
<th>S</th>
<th>CA</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>1.56</td>
<td>1.38</td>
<td>13.27</td>
<td>1.32</td>
<td>1.22</td>
<td>1.19</td>
<td></td>
</tr>
<tr>
<td>CN</td>
<td>21.52</td>
<td>0.68</td>
<td>22.75</td>
<td>0.68</td>
<td>0.75</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>CS</td>
<td>21.49</td>
<td>0.05</td>
<td>22.54</td>
<td>0.68</td>
<td>0.74</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>SA</td>
<td>21.15</td>
<td>28.39</td>
<td>19.48</td>
<td>17.37</td>
<td>14.08</td>
<td>8.01</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>21.51</td>
<td>0.05</td>
<td>0.00</td>
<td>22.54</td>
<td>0.76</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>CA</td>
<td>21.71</td>
<td>5.77</td>
<td>9.58</td>
<td>22.69</td>
<td>9.95</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td>SI</td>
<td>20.28</td>
<td>10.71</td>
<td>10.73</td>
<td>15.37</td>
<td>10.62</td>
<td>10.59</td>
<td></td>
</tr>
</tbody>
</table>

Figure 17 shows the probability of line congestion in different hours of the day. This plot highlights that lines 4 (Mainland to SI) and 21 (N to CN) are congested in over 60% of the time in peak-load hours.30

30 Line 37 denotes the link between the North zone and the virtual Swiss zone.
C Market Clearing

The Power Exchange’s (PXE) objective function is to maximize net welfare accounting for transmission grid constraints. Market participants submit an offer curve and/or a demand curve, respectively. Given an aggregate supply function \( \theta_o = (b_o, g_o) \) and an aggregate demand function \( \theta_b = (b_b, g_b) \), the PXE’s problem reads

\[
\begin{align*}
\text{maximize} & \quad b_b^T x_b - b_o^T x_o \\
\text{subject to} & \quad 1^T x_b = 1^T x_o \\
& \quad \sum_{z \in Z} S_{kl}^z (1^T x_o^z - 1^T x_b^z) \leq f_{kl}, \ (k, l) \in A \\
& \quad x_k \leq g_k, \ k \in \{b, o\} \\
& \quad x_k \in \mathbb{R}_0^+, \ k \in \{b, o\}. 
\end{align*}
\]

The values of \( x \) that maximize (6a) have to be balanced, i.e., demand equals supply (Equation 6b). Furthermore, these quantities must satisfy the network feasibility constraints, that is, the resulting power flows should not exceed the thermal limits \( f_{kl} \) of the transmission lines. The matrix \( S \) in Equation 6c gives the contribution of a net injection in zone \( z \) to transmission line \( (k, l) \in A \), where the set \( A \) contains all transmission lines of the system. Market-clearing quantities \( x \) must be larger equal than zero and less equal than their submitted levels \( (g) \). Let \( \lambda \) be the dual multiplier of the balance constraint (6b) and \( \mu_{kl} \) the duals of the transmission constraints in (6c), then the optimal zonal prices are given as

\[
p_z = \lambda - \sum_{(k, l) \in A} S_{kl}^z \mu_{kl}, \quad \forall z \in Z. \tag{7}
\]

Needless to say that if all the constraints in (6c) are not binding, their multipliers \( (\mu_{kl}, (k, l) \in A) \) would be zero and hence the market clearing price is the same in each zone.
C.1 Uniform Purchase Price

The Italian day-ahead market for electricity has one peculiarity, which is that the demand side bidders face a uniform purchase price (UPP). The historical reason for this rule is that consumers on remote islands or the poorer South should not be disadvantaged. From a modeling perspective this implies that the following additional constraint needs to hold

\[ \bar{p} \sum_{r \in B_R} x_r = \sum_{z \in Z} \sum_{r \in B_r(z)} p_z x_r, \]  

(8)

where \( B_R \) is the subsets of demand bids containing only regular demand bids. Regular demand bids originate from a domestic zone and are not from a pumped-hydro storage unit. (8) can be interpreted as money conservation law. On the ride hand side of (8) we could also sum over all offers. However, incorporating (8) into Problem 6 renders the problem non-linear. There are three possibilities, to extract the UPP: (i) Search for an optimal solution that fulfills (8) by solving the problem for different UPP candidates,\(^{31}\) (ii) set-up the problem as a bi-level problem which can be translated into a single level problem and solved with off-the-shelf mixed integer programming solvers (see, e.g., Savelli et al., 2017), or, (iii) calculate the UPP as the demand quantity weighted zonal price. In this paper, we went for the last option because of the accuracy of this simple approach (see Section C.4).

C.2 Price/Quantity Indeterminacy

Market participants submit step-functions. Hence, rules that define what to do when aggregated supply and demand step-functions overlap are necessary. The rules applied by PXE are the following: In case supply and demand curves intersect at a vertical portion of both curves, the market-clearing solution is where allocated quantity is maximized. In case supply and demand curves intersect at a horizontal portion of both curves the market-clearing

\(^{31}\)This iterative approach is the method used by the Italian power exchange. A more detailed description of the algorithm can be found here: [http://www.mercatoelettrico.org/En/MenuBiblioteca/Documenti/20041206UniformPurchase.pdf](http://www.mercatoelettrico.org/En/MenuBiblioteca/Documenti/20041206UniformPurchase.pdf).
solution is where the price is minimal. Practically this is archived by shifting the curves by a small amount.\footnote{More details can be found under: \url{http://www.mercatoelettrico.org/En/MenuBiblioteca/Documenti/20100429MarketSplitting.pdf}.

32}

C.3 Uniqueness

The offer data come with a merit order number which is used to prioritize amongst marginal supply and demand bids. For marginal supply/demand bid prices that are the same, the merit order number provides a ranking to these bids. According to the Decision 111/06, Article 30.7, published by the Italian Regulatory Authority for Electricity Gas and Water, the following priority is considered in case the same price is offered: offers from essential units, offers from non-programmable renewables, combined heat and power plants, incentivized power plants (e.g., CIP-6/92), power plants using national fuels, and then all other plants. In cases where offers/bids have equal priority, the chronological order of receipt of bids and offers acts as a tie-breaker (first in, first out).

C.4 Accuracy

We are able to solve the Italian Day-Ahead market of electricity with almost perfect accuracy. Table 9 shows the mean absolute deviation between the actual zonal prices and the replicated zonal prices in the domestic demand zones and the mean absolute deviation between the actual and replicated uniform purchase price (UPP). The small deviations result from not endogenously deriving the UPP.\footnote{We also implemented the UPP search procedure applied by the power exchange and described in Section C.1. However, we decided to trade this slight inaccuracy against a much faster market clearing algorithm.} The replicated clearing spans the period 2007-09-01 to 2007-11-01.
Table 9: Mean Absolute Deviation of Actual vs. Replicated Prices

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C.5 Simplified Topology

From Figure 16 and Table 8, we conclude that congestion is mainly an issue between the North zone and the Center-North zone, as well as between the Calabrian zone and the Sicily zone. Furthermore, the virtual trading zone with Switzerland is to be congested frequently. This inspired us to reduce the problem complexity by using a simpler three-national-zone network and also model the Swiss zone and its connecting zone separately. More precisely, the simplified network topology consists of: (i) the North zone which includes virtual foreign trading zones except the Swiss zone and its connecting zone, (ii) the Center zones which covers all national zones on the mainland except the North zone plus Sardinia and the
virtual trading zone with Greece, and (iii) Sicily. We refer to these zones as North (N), Center (C), and Sicily (SI).

Clearing the market with the underlying simpler network captures the clearing prices and quantities sufficiently well. The congestion pattern for the relevant period essentially does not change and the average uniform purchase price is 107.12 EUR as compared to 107.02 EUR with the original clearing.

D Calculating Residual Demand Hyper-Surfaces

The market-clearing algorithm described in Problem 6 gives zonal equilibrium prices for any vector of a strategic firm’s zonal output holding demand bids and competitors’ offers fix. We operationalize the procedure to derive residual demand hyper-surface by defining an output grid for each of the two strategic firms and solve the market-clearing for any zonal output combinations on the grid. The zonal output values of each strategic firm together with the resulting zonal market-clearing prices define then the residual demand hyper-surfaces.

E Counter-Intuitive Effects of Neglecting Transmission Constraints on Residual Demand

In Section 3.2, we show that a firm’s residual demand will become less steep when ignoring transmission constraints and therefore it will offer more aggressively. This view appears to be consistent with common wisdom that in a larger pool less unilateral market power potential exists. However, we also want to highlight a phenomenon that can happen when a firm is pivotal in one zone but not overall. Assume a two-zone network with a strategic firm that has capacity in both zones. Assume further a situation where the transmission constraint between the two zones is binding and that the firm is pivotal in one zone but not in the other. We depict the stylized residual demand curves describing this case in Figure 18,
Panel (a) and (b). The first horizontal step in Panel (b) should indicate the demand at the price-cap. The firm’s pivotalness in the high-price market will ensure that the firm will offer at least the part of its zonal capacity the firm is pivotal. Because the transmission constraint is binding between the two zones, the firm’s residual demand curve in the low-price market (Panel a) consists also of some bids from the firm’s opponents that are “cut-off” (left of the vertical axis).

In Panel (c), we show how the firm’s residual demand curve would appear if the firm were to ignore transmission constraints in the market-clearing. We can observe that in such a case the horizontal part of the residual demand curve in the high price market would be replaced by the “cut-off” bids left to the y-axis in the low price market. In other words, by assuming transmission constraints away the firm’s pivotalness in the high price zone would vanish, hence, a perfectly elastic part of the zonal residual demand will be traded against a less elastic residual demand part flowing in from originally “cut-off” bids. As a consequence the hypothetical residual demand curve might even be steeper for a small amount of output than the actual residual demand surfaces. Consequently, the firm may offer less aggressively if it were to ignore transmission constraints.

In Figure 5, Panel (b), we observe such a case with a firm being pivotal for about 1 GW of its capacity. This carries over to the uniform purchase price – the load share weighted average zonal price – which is the relevant price for the demand side of the market as can be seen in Panel (c). In Figure 12, Panel (b), the uniform purchase price is depicted conditional on different supply in different zones. If the firm would not take transmission constraints into account when offering, its hypothetical residual demand curve would look like in Figure 12, Panel (a).

[Figure 18 about here.]

[Figure 19 about here.]
F  Best-Response Offer Function Calculation

The best-response offer problem for supplier $i$ finds the value of $\theta_i$, the vector of offer prices and quantities for all generation units owned by the firm $i$, to maximize the expected value firm $i$'s variable profit function given in (2). The expectation of this variable profit function is taken with respect to the distribution of residual demand surfaces faced by firm $i$. Each possible residual demand surface realization for a given value $\theta_i$ gives rise to a set of zonal prices and generation unit-level dispatch quantities for all generation units through the locational market-clearing mechanism. For the case of Italy, the market-clearing mechanism takes the bid and offer curves of market participants and solves the optimization problem given in Section C.

The resulting prices and dispatch levels from the market-clearing optimization problem are substituted into (2) to obtain the realized variable profits for that residual demand surface realization. Repeating this process for all possible residual demand surface realizations and a given value $\theta_i$ and averaging these realized variable profit values yields the expected profit for firm $i$ at $\theta_i$. Finding the value of $\theta_i$ that maximizes this function yields firm $i$'s expected profit-maximizing offer curves for all of its generation units.

In order to facilitate readability of the mathematical presentation of the solution to this problem we omit the fact that for each value of $\theta_i$ the market-clearing problem must be solved for each residual demand surface in order to determine the zonal prices and generation unit-level dispatch quantities necessary to compute the realized variable profit of firm $i$ for that residual demand surface realization. For our choice of 20 residual demand surface scenarios, this logic implies that we must solve 20 market-clearing problems each time we compute the expected profit function for firm $i$ for given value of $\theta_i$. The solutions to these 20 market-clearing problems are formulated in terms of Karush-Kuhn-Tucker (KKT) conditions, which implies the that firm’s expected profit maximization problem is a mathematical program with equilibrium constraints (MPEC).
The PXE’s market-clearing necessary Karush-Kuhn-Tucker (KKT) conditions are

\begin{align}
\mathbf{b}_i - W_i \mathbf{p} - \overline{\mathbf{\mu}}_i + \overline{\mathbf{\mu}}_i &= 0, \\
\mathbf{b}_{-i} - W_{-i} \mathbf{p} - \overline{\mathbf{\mu}}_{-i} + \overline{\mathbf{\mu}}_{-i} &= 0, \\
-\mathbf{b}_b + W_b \mathbf{p} - \overline{\mathbf{\mu}}_b + \overline{\mathbf{\mu}}_b &= 0, \\
1^T \mathbf{x}_b - 1^T \mathbf{x}_{-i} - 1^T \mathbf{x}_i &= 0, \\
\sum_{z \in Z} S_{kl}^z \left(1^T \mathbf{x}_i^z + 1^T \mathbf{x}_{-i}^z - 1^T \mathbf{x}_b^z\right) - f_{kl} &\leq 0, \ (k, l) \in A,
\end{align}

\begin{align}
0 &\leq \mathbf{x}_i \perp \overline{\mathbf{\mu}}_i \geq 0, \\
0 &\leq \mathbf{g}_i - \mathbf{x}_i \perp \overline{\mathbf{\mu}}_i \geq 0, \\
0 &\leq \mathbf{x}_{-i} \perp \overline{\mathbf{\mu}}_{-i} \geq 0, \\
0 &\leq \mathbf{g}_{-i} - \mathbf{x}_{-i} \perp \overline{\mathbf{\mu}}_{-i} \geq 0, \\
0 &\leq \mathbf{x}_b \perp \overline{\mathbf{\mu}}_b \geq 0, \\
0 &\leq \mathbf{g}_b - \mathbf{x}_b \perp \overline{\mathbf{\mu}}_b \geq 0, \\
0 &\leq f_{kl} - \sum_{z \in Z} S_{kl}^z \left(1^T \mathbf{x}_i^z + 1^T \mathbf{x}_{-i}^z - 1^T \mathbf{x}_b^z\right) \perp \mu_{kl} \geq 0, \ (k, l) \in A,
\end{align}

whereas the matrix $W$ maps bids to zones, $\mathbf{p}$ is the vector of zonal prices, and $\overline{\mathbf{\mu}}(\cdot)$ and $\mathbf{\mu}(\cdot)$ are the dual variables of (6d) and (6c) respectively. Note further, that we split $\mathbf{\theta}_o = (\mathbf{b}_o, \mathbf{g}_o)$ into $\mathbf{\theta}_i = (\mathbf{b}_i, \mathbf{g}_i)$ and $\mathbf{\theta}_{-i} = (\mathbf{b}_{-i}, \mathbf{g}_{-i})$. The PXE’s program is a strictly concave-maximization problem, so the KKT conditions are also sufficient.\(^{35}\)

In a second step we rewrite the firm’s optimization problem defined in (4) and include the single KKT conditions as constraints, although for the actual problem solve there are 20 sets of KKT conditions, one for residual demand surface realization. Firm $i$’s expected

\(^{34}\)Note that we minimize negative net-welfare instead of maximizing net-welfare.

\(^{35}\)Note that we have implemented all mathematical programs accounting for uniqueness as described in Sections C.2 and C.3. We decided to omitted these additional constants in the notation to keep it easier to follow.
profit function is the sample average of 20 variable profit function values associated with the
20 residual demand surface realizations.

\[
\begin{align*}
\text{minimize} & \quad Q^R_i \bar{p} + \bar{p} \sum_z QC'_{iz} + \sum_z c^T_{iz} x_{iz} - \sum_z (1^T x_{iz})p_z \\
\text{subject to} & \quad 0 \leq b_i \leq \hat{b}_i \quad (10a) \\
& \quad 0 \leq g_i \leq \hat{g}_i \quad (10b) \\
& \quad (7), (9a)-(9l). \quad (10c)
\end{align*}
\]

Note that firm \(i\)'s offer price vector entries in \(b_i\) are constrained by the PXE's price floor (zero) and price cap (\(\hat{b}\)). Furthermore, the offer quantities are supposed to be greater equal than zero and below a maximum value (\(\hat{g}\)). The short run variable cost of production \(c\) are assumed to be constant for each unit.

Given that \(Q^R, QC, c,\) and \(w\) - the vector of zonal load weights that determine \(\bar{p}\)-are exogenous, the only non-linear term in (10a) is \(\sum_z (1^T x_{iz})p_z\). In order to get rid of the non-linear term in the objective function, we make use of the strong duality theorem. Since the PXE’s problem 6 is convex we know that the function value of the dual is equal to the function value of the primal problem at the optimum. Hence, we are allowed to equalize the objective function of the primal problem with the objective function of the dual problem which results in

\[
b^T_i x_i + b^{T}_{-i} x_{-i} - b^T_b x_b = -\mu^T_i g_i - \mu^T_{-i} g_{-i} - \mu^T_b g_b - \mu^T f.\quad (11)
\]

\(36\)Equation (10c) can also easily be replaced by capping the production of a unit by its maximum available capacity as we defined it in Equation 3. However, since we solve the problem by holding \(g_i\) fix, we decided to economize on notation.
Furthermore, we take the inner product between (9a) and $x_i$, which yields

$$b_i^T x_i - (W_i p)^T x_i - \mu_i^T x_i + \overline{\mu}_i^T x_i = 0.$$  \hspace{1cm} (12)

From (9f) we know that $\mu_i^T x_i = 0$ and according to (9g), $\overline{\mu}_i^T x_i = \overline{\mu}_i^T g_i$. Using this, and plugging (12) into (11) gives us

$$(W_i p)^T x_i = b_i^T x_b - b_{-i}^T x_{-i} - \overline{\mu}_{-i}^T g_{-i} - \overline{\mu}_b^T g_b - \mu^T f.$$  \hspace{1cm} (13)

Finally, we can replace the non-linear term $\sum_z (1^T x_{iz}) p_z$ in (10a) with its linear equivalent defined in (13). Note that $\sum_z (1^T x_{iz}) p_z$ and $(W_i p)^T x_i$ are equivalent.
Figure 1: Market-Clearing in Transport Capacity Constrained Symmetric Markets

Panel (a): Uncongested market, Panel (b): Congested markets.
Figure 2: Inverse Residual Demand Surfaces in Symmetric Markets

Figure 3: Market Outcomes in Transport-Capacity-Constrained Asymmetric Markets

Panel (a): Uncongested market, Panel (b): Congested markets.
Figure 4: Inverse Residual Demand Surfaces in Asymmetric Markets

Figure 5: Inverse Residual Demand Surfaces for Incumbent in a Peak Hour

The figures show the incumbent’s inverse residual demand surfaces in the North zone (Panel a), Center zone (Panel b) and the uniform purchase price (Panel c), on September 21, 2007, Hour 11. The supply in Sicily is held constant at the level of dispatch at actual offer curves. Only relevant capacity offered at prices above short-term variable cost considered. Quantities are in GW and prices in EUR/MWh.
The figures show the main competitor’s inverse residual demand surfaces in the North zone (Panel a), Center zone (Panel b) and the uniform purchase price (Panel c), on September 21, 2007, Hour 11. The supply in Sicily is held constant at the level of dispatch at actual offer curves. Only thermal capacity excluding self-schedules considered. Quantities are in GW and prices in EUR/MWh.
Figure 7: Zonal Inverse Residual Demand Curves for Incumbent in a Peak Hour Conditional on Supply in Other Zones

(a)  
(b)  
(c)  

The figures show the incumbent’s inverse residual demand function conditional on supply in the North zone (Panel a), Center zone (Panel b), and the Sicily zone (Panel c), on September 21, 2007, Hour 11. Only relevant capacity offered at prices above short-term variable cost considered. Quantities are in GW and prices in EUR/MWh.
Figure 8: Zonal Inverse Residual Demand Curves Conditional on Demand in Another Zone

The figures show the incumbent’s inverse residual demand function conditional on the level of demand in the North zone (Panel a) and Center zone (Panel b), on September 21, 2007, Hour 11. Only relevant capacity offered at prices above short-term variable cost considered. Quantities are in GW and prices in EUR/MWh.
The figures show the incumbent’s (Panel a) and the main competitor’s (Panel b) unilateral congestion power defined as a zonal price difference between the Center zone and the North zone, on September 21, 2007, Hour 11. The supply in Sicily is held constant at the level of dispatch at actual offer curves. In case of the incumbent only relevant capacity offered at prices above short-term variable cost considered, in case of the main competitor only thermal capacity excluding self-schedules considered. Quantities are in GW and prices in EUR/MWh.
Figure 10: Market-Clearing in a Network

Market clearing of September 21, Hour 11, accounting for flows between zones. Panel (a) North zone, Panel (b) Center zone. Quantities are in GW and prices in EUR/MWh.
Hypothetical single price market-clearing September 21, Hour 11. Quantities are in GW and prices in EUR/MWh.
Figure 12: Hypothetical Inverse Residual Demand Curve vs. Uniform Purchase Price for Incumbent in a Peak Hour

(a) & (b)

The figures show the incumbent’s residual demand function ignoring transmission constraints (Panel a) and the uniform purchase price function conditional on supply in different zones (Panel b), on September 21, 2007, Hour 11. The supply in Sicily as well as the net-imports are held constant at the level of dispatch at actual offer curves. Only relevant capacity offered at prices above short-term variable cost considered. Quantities are in GW and prices in EUR/MWh.
Figure 13: Incumbent’s Optimal Zonal Offer Curves

September 4, Hour 12. Panel (a): North; Panel (b): Center, Panel (c): Sicily. Only relevant units. Quantities are in GW and prices in EUR/MWh.
Figure 14: Main Competitor’s Optimal Zonal Offer Curves

September 25, Hour 12. Panel (a): North; Panel (b): Center, Panel (c): Sicily. Only relevant units. Quantities are in GW and prices in EUR/MWh.
Figure 15: Transmission Network Topology in 2007
Figure 16: Zonal Day-Ahead Market Price Distributions
Figure 17: Spatial and Temporal Congestion Shares
Figure 18: Residual Demand Curves in Stylized Connected Zonal Markets

Figures show firm $i$’s residual demand $RD_i$ in two connected zonal markets (Panel a, and Panel b) with a binding transmission constraint. Panel (c): Both markets combined.
Figure 19: Hypothetical Inverse Residual Demand Curve for Main Competitor in a Peak Hour

The figure shows the main competitor’s residual demand curve ignoring transmission constraints, on September 21, 2007, Hour 11. Only thermal capacity excluding self-schedules considered. Quantities are in GW and prices in EUR/MWh.